CS420, Machine Learning, Lecture 7

Ranking and Filtering

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http://wnzhang.net/teaching/cs420/index.html

Content of This Course

- Another ML problem: Ranking
  - Learning to rank
  - Pointwise methods
  - Pairwise methods
  - Listwise methods

- A data mining application: Recommendation
  - Overview
  - Collaborative filtering
  - Rating prediction
  - Top-N ranking

Ranking Problem

Learning to rank
Pointwise methods
Pairwise methods
Listwise methods

Sincerely thank Dr. Tie-Yan Liu

Regression and Classification

- Supervised learning

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \mathcal{L}(y_i, f_\theta(x_i))$$

- Two major problems for supervised learning
  - Regression

$$\mathcal{L}(y_i, f_\theta(x_i)) = \frac{1}{2}(y_i - f_\theta(x_i))^2$$

  - Classification

$$\mathcal{L}(y_i, f_\theta(x_i)) = -y_i \log f_\theta(x_i) - (1 - y_i) \log(1 - f_\theta(x_i))$$

Learning to Rank Problem

- Input: a set of instances

$$X = \{x_1, x_2, \ldots, x_n\}$$

- Output: a rank list of these instances

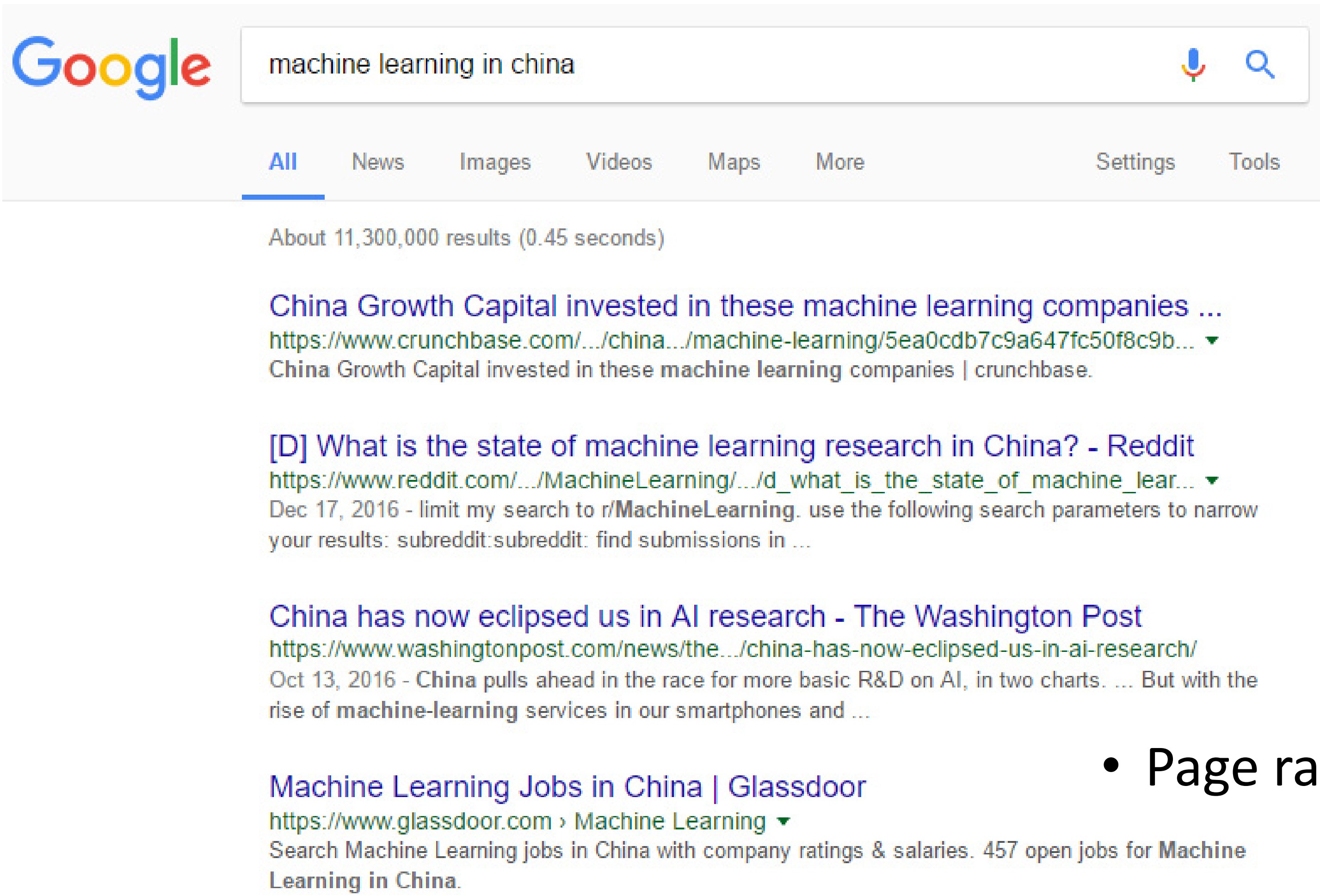
$$\hat{Y} = \{x_{r_1}, x_{r_2}, \ldots, x_{r_n}\}$$

- Ground truth: a correct ranking of these instances

$$Y = \{x_{y_1}, x_{y_2}, \ldots, x_{y_n}\}$$

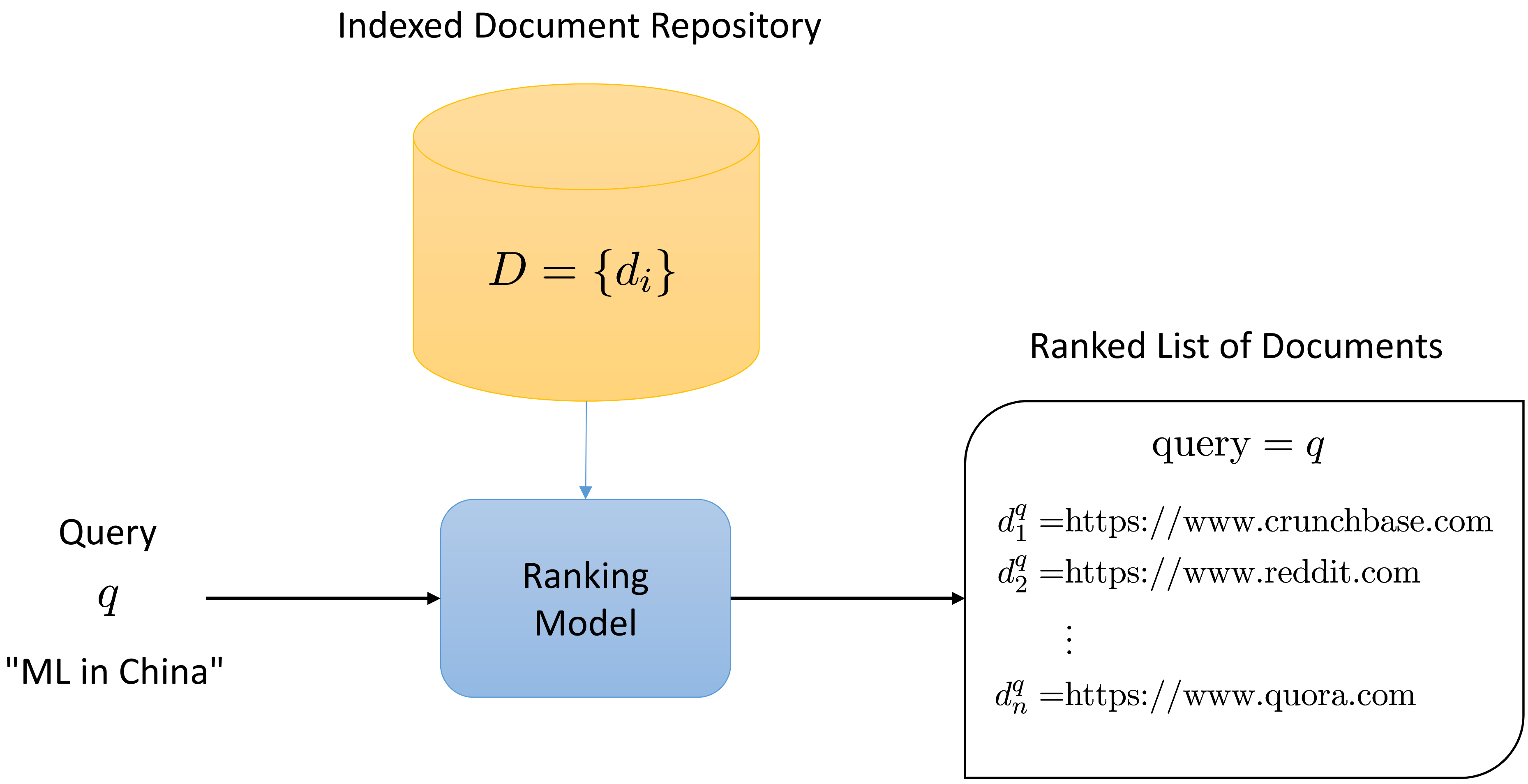
A Typical Application

- Webpage ranking given a query



- Page ranking

Webpage Ranking



Model Perspective

- In most existing work, learning to rank is defined as having the following two properties
  - Feature-based
    - Each instance (e.g. query-document pair) is represented with a list of features
  - Discriminative training
    - Estimate the relevance given a query-document pair
    - Rank the documents based on the estimation

$$y_i = f_\theta(x_i)$$

Learning to Rank

- Input: features of query and documents
  - Query, document, and combination features
- Output: the documents ranked by a scoring function

$$y_i = f_\theta(x_i)$$

- Objective: relevance of the ranking list
  - Evaluation metrics: NDCG, MAP, MRR…
- Training data: the query-doc features and relevance ratings

Training Data

- The query-doc features and relevance ratings

Query='ML in China'

Features

Rating	Document	Query Length	Doc PageRank	Doc Length	Title Rel.	Content Rel.
3	d_1=http://crunchbase.com	0.30	0.61	0.47	0.54	0.76
5	d_2=http://reddit.com	0.30	0.81	0.76	0.91	0.81
4	d_3=http://quora.com	0.30	0.86	0.56	0.96	0.69

Query features

Document features

Query-doc features

Learning to Rank Approaches

- Learn (not define) a scoring function to optimally rank the documents given a query

- Pointwise
    - Predict the absolute relevance (e.g. RMSE)
- Pairwise
    - Predict the ranking of a document pair (e.g. AUC)
- Listwise
    - Predict the ranking of a document list (e.g. Cross Entropy)

Tie-Yan Liu. Learning to Rank for Information Retrieval. Springer 2011.
http://www.cda.cn/uploadfile/image/20151220/20151220115436_46293.pdf

Pointwise Approaches

- Predict the expert ratings
  - As a regression problem

$$y_i = f_\theta(x_i)$$

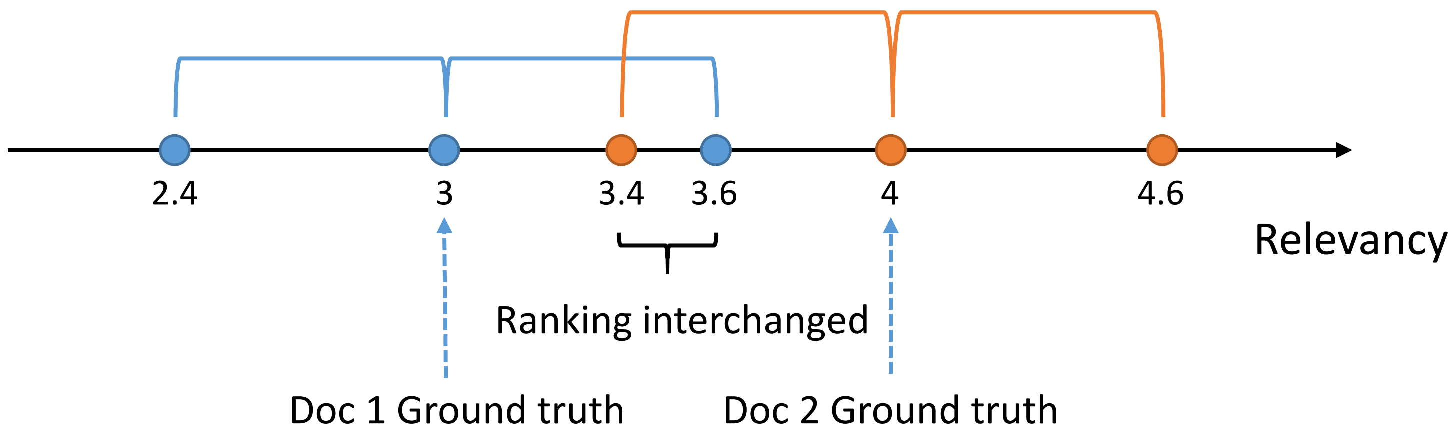
$$\min_\theta \frac{1}{2N} \sum_{i=1}^{N} (y_i - f_\theta(x_i))^2$$

Query='ML in China'

Features

Rating	Document	Query Length	Doc PageRank	Doc Length	Title Rel.	Content Rel.
3	d_1=http://crunchbase.com	0.30	0.61	0.47	0.54	0.76
5	d_2=http://reddit.com	0.30	0.81	0.76	0.91	0.81
4	d_3=http://quora.com	0.30	0.86	0.56	0.96	0.69

Point Accuracy != Ranking Accuracy



- Same square error might lead to different rankings

Pairwise Approaches

- Not care about the absolute relevance but the relative preference on a document pair

- A binary classification

$$q^{(i)}$$

$$\begin{bmatrix} d_1^{(i)}, 5 \\ d_2^{(i)}, 3 \\ \vdots \\ d_{n^{(i)}}^{(i)}, 2 \end{bmatrix} \xrightarrow{\text{Transform}} \begin{array}{c} q^{(i)} \\ \left\{ \underset{5>3}{(d_1^{(i)}, d_2^{(i)})}, \underset{5>2}{(d_1^{(i)}, d_{n^{(i)}}^{(i)})}, \ldots, \underset{3>2}{(d_2^{(i)}, d_{n^{(i)}}^{(i)})} \right\} \end{array}$$

Binary Classification for Pairwise Ranking

- Given a query q and a pair of documents (d_i, d_j)
- Target probability $y_{i,j} = \begin{cases} 1 & \text{if } i \triangleright j \\ 0 & \text{otherwise} \end{cases}$
- Modeled probability

$$P_{i,j} = P(d_i \triangleright d_j | q) = \frac{\exp(o_{i,j})}{1 + \exp(o_{i,j})}$$

$$o_{i,j} \equiv f(x_i) - f(x_j)$$

x_i is the feature vector of (q, d_i)

- Cross entropy loss

$$\mathcal{L}(q, d_i, d_j) = -y_{i,j} \log P_{i,j} - (1 - y_{i,j}) \log(1 - P_{i,j})$$

RankNet

- The scoring function $f_\theta(x_i)$ is implemented by a neural network
- Modeled probability $P_{i,j} = P(d_i \rhd d_j | q) = \frac{\exp(o_{i,j})}{1 + \exp(o_{i,j})}$

$$o_{i,j} \equiv f(x_i) - f(x_j)$$

- Cross entropy loss

$$\mathcal{L}(q, d_i, d_j) = -y_{i,j} \log P_{i,j} - (1 - y_{i,j}) \log(1 - P_{i,j})$$

- Gradient by chain rule

BP in NN

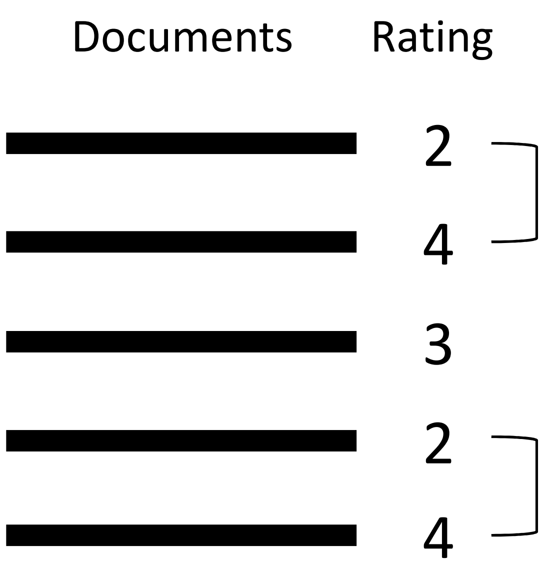
$$\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial \theta} = \frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial P_{i,j}} \frac{\partial P_{i,j}}{\partial o_{i,j}} \frac{\partial o_{i,j}}{\partial \theta}$$

$$= \frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial P_{i,j}} \frac{\partial P_{i,j}}{\partial o_{i,j}} \Big(\frac{\partial f_\theta(x_i)}{\partial \theta} - \frac{\partial f_\theta(x_j)}{\partial \theta} \Big)$$

Burges, Christopher JC, Robert Ragno, and Quoc Viet Le. "Learning to rank with nonsmooth cost functions." *NIPS*. Vol. 6. 2006.

Shortcomings of Pairwise Approaches

- Each document pair is regarded with the same importance



Same pair-level error but different list-level error

Ranking Evaluation Metrics

- For binary labels $y_i = \begin{cases} 1 & \text{if } d_i \text{ is relevant with } q \\ 0 & \text{otherwise} \end{cases}$

- Precision@*k* for query *q*

$$P@k = \frac{\#\{\text{relevant documents in top } k \text{ results}\}}{k}$$

- Average precision for query *q*

$$AP = \frac{\sum_k P@k \cdot y_{i(k)}}{\#\{\text{relevant documents}\}}$$

1
0
1
0
1

  - *i*(*k*) is the document id at *k*-th position

$$AP = \frac{1}{3} \cdot \left(\frac{1}{1} + \frac{2}{3} + \frac{3}{5}\right)$$

- Mean average precision (MAP): average over all queries

Ranking Evaluation Metrics

- For score labels, e.g.,

$$y_i \in \{0, 1, 2, 3, 4\}$$

- Normalized discounted cumulative gain (NDCG@*k*) for query *q*

$$NDCG@k = Z_k \sum_{j=1}^{k} \frac{2^{y_{i(j)}} - 1}{\log(j+1)}$$

Gain

Discount

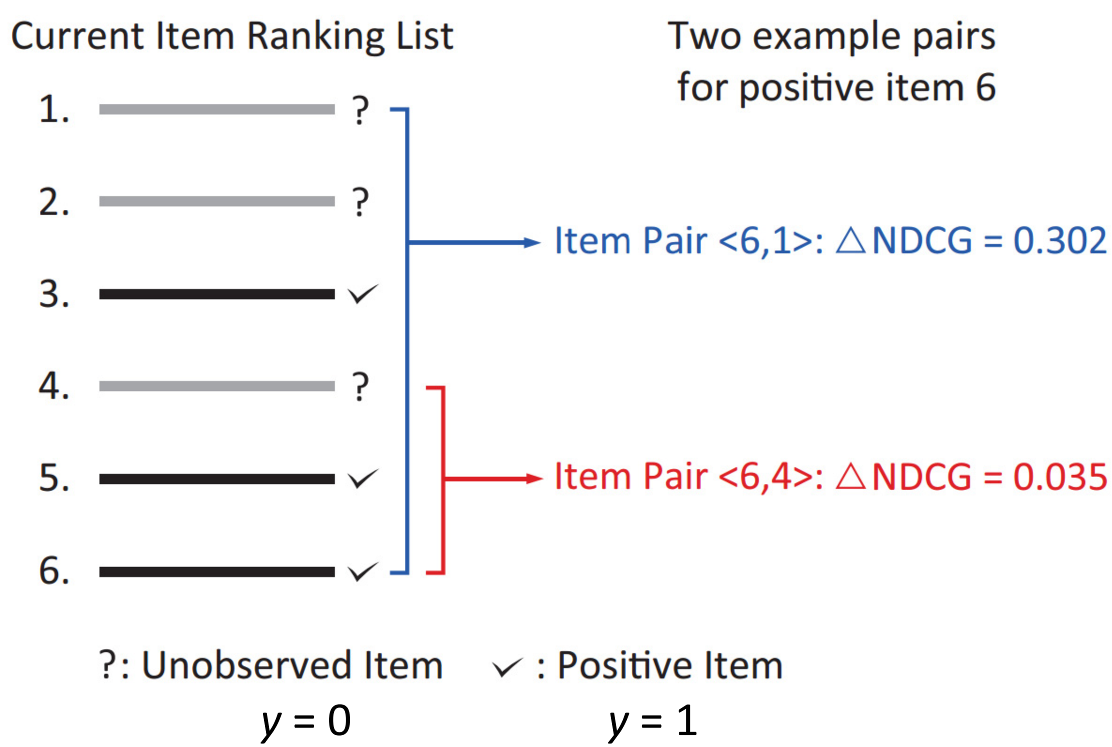
Normalizer

  - *i*(*j*) is the document id at *j*-th position
  - Z_k is set to normalize the DCG of the ground truth ranking as 1

Shortcomings of Pairwise Approaches

- Same pair-level error but different list-level error

$$NDCG@k = Z_k \sum_{j=1}^{k} \frac{2^{y_{i(j)}} - 1}{\log(j+1)}$$



Listwise Approaches

- Training loss is directly built based on the difference between the prediction list and the ground truth list
- Straightforward target
  - Directly optimize the ranking evaluation measures
- Complex model

ListNet

- Train the score function $y_i = f_\theta(x_i)$
- Rankings generated based on $\{y_i\}_{i=1\ldots n}$
- Each possible *k*-length ranking list has a probability

$$P_f([j_1, j_2, \ldots, j_k]) = \prod_{t=1}^{k} \frac{\exp(f(x_{j_t}))}{\sum_{l=t}^{n} \exp(f(x_{j_l}))}$$

- List-level loss: cross entropy between the predicted distribution and the ground truth

$$\mathcal{L}(\boldsymbol{y}, f(\boldsymbol{x})) = -\sum_{g \in \mathcal{G}_k} P_y(g) \log P_f(g)$$

- Complexity: many possible rankings

Cao, Zhe, et al. "Learning to rank: from pairwise approach to listwise approach." *Proceedings of the 24th international conference on Machine learning*. ACM, 2007.

Distance between Ranked Lists

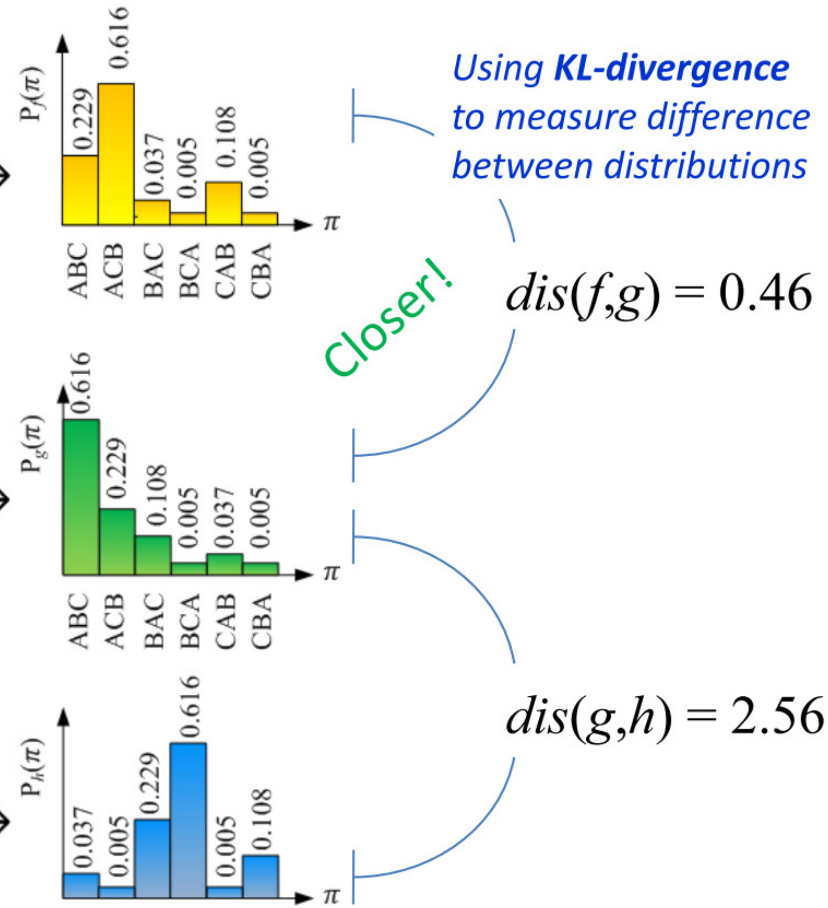
- A similar distance: KL divergence

$\varphi = exp$

f: $f(A) = 3, f(B)=0, f(C)=1$;
Ranking by f: ABC $\leftrightarrow$

g: $g(A) = 6, g(B)=4, g(C)=3$;
Ranking by g: ABC $\leftrightarrow$

h: $h(A) = 4, h(B)=6, h(C)=3$;
Ranking by h: ACB $\leftrightarrow$



Pairwise vs. Listwise

- Pairwise approach shortcoming
  - Pair-level loss is away from IR list-level evaluations
- Listwise approach shortcoming
  - Hard to define a list-level loss under a low model complexity

- A good solution: LambdaRank
  - Pairwise training with listwise information

Burges, Christopher JC, Robert Ragno, and Quoc Viet Le. "Learning to rank with nonsmooth cost functions." *NIPS*. Vol. 6. 2006.

LambdaRank

- Pairwise approach gradient

$$o_{i,j} \equiv f(x_i) - f(x_j)$$

$$\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial \theta} = \underbrace{\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial P_{i,j}} \frac{\partial P_{i,j}}{\partial o_{i,j}}}_{\lambda_{i,j}} \Big(\frac{\partial f_\theta(x_i)}{\partial \theta} - \frac{\partial f_\theta(x_i)}{\partial \theta} \Big)$$

Pairwise ranking loss

Scoring function itself

Current ranking list

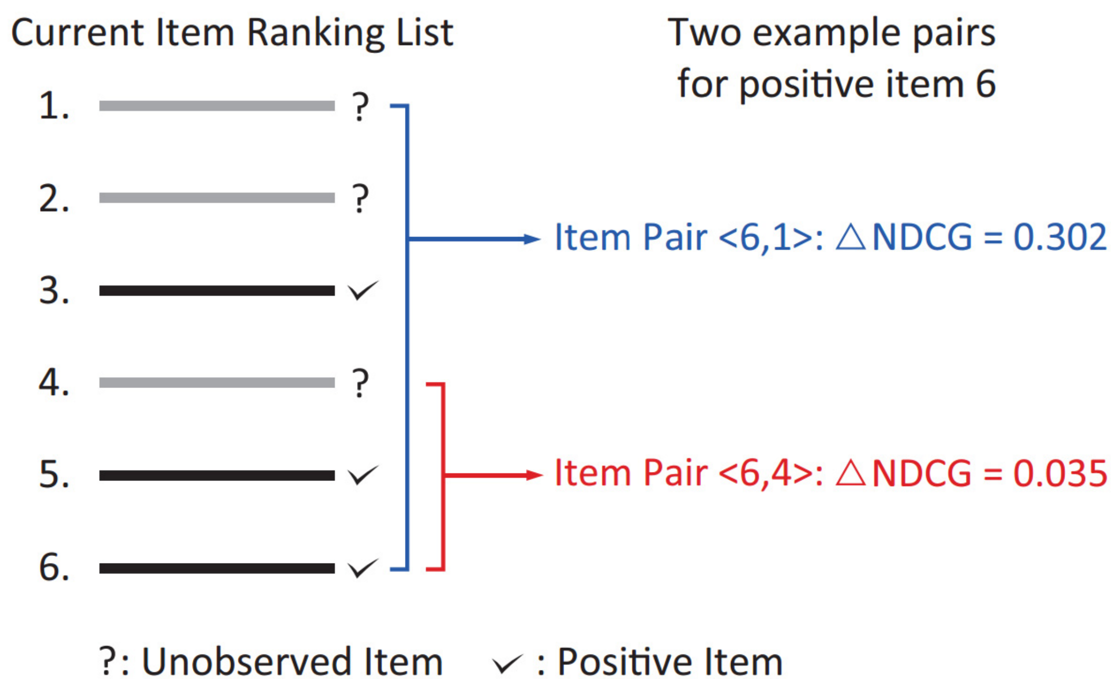
- LambdaRank basic idea
  - Add listwise information into $\lambda_{i,j}$ as $h(\lambda_{i,j}, g_q)$

$$\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial \theta} = h(\lambda_{i,j}, g_q) \Big(\frac{\partial f_\theta(x_i)}{\partial \theta} - \frac{\partial f_\theta(x_j)}{\partial \theta} \Big)$$

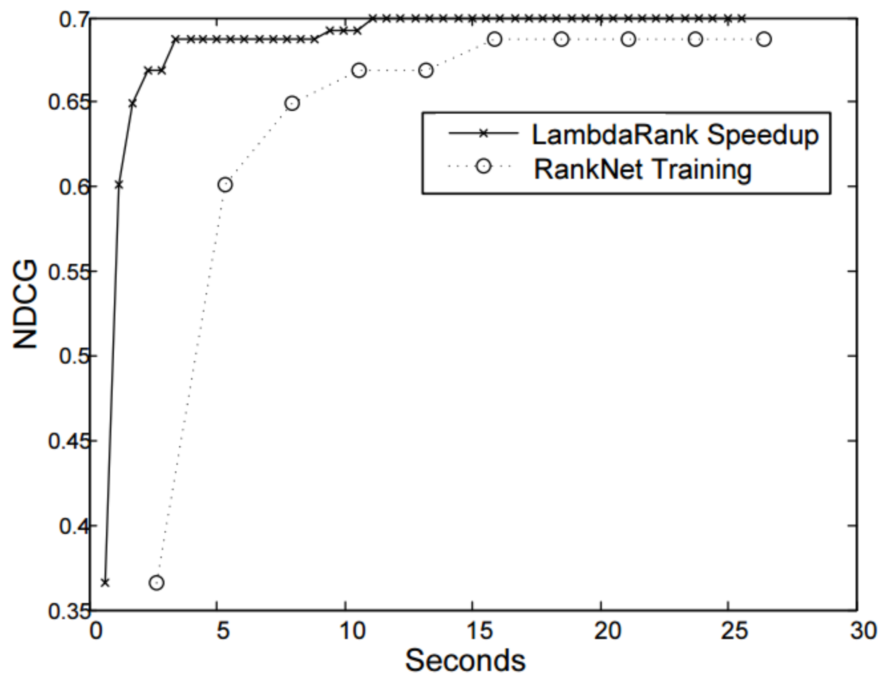
LambdaRank for Optimizing NDCG

- A choice of Lambda for optimize NDCG

$$h(\lambda_{i,j}, g_q) = \lambda_{i,j} \Delta NDCG_{i,j}$$



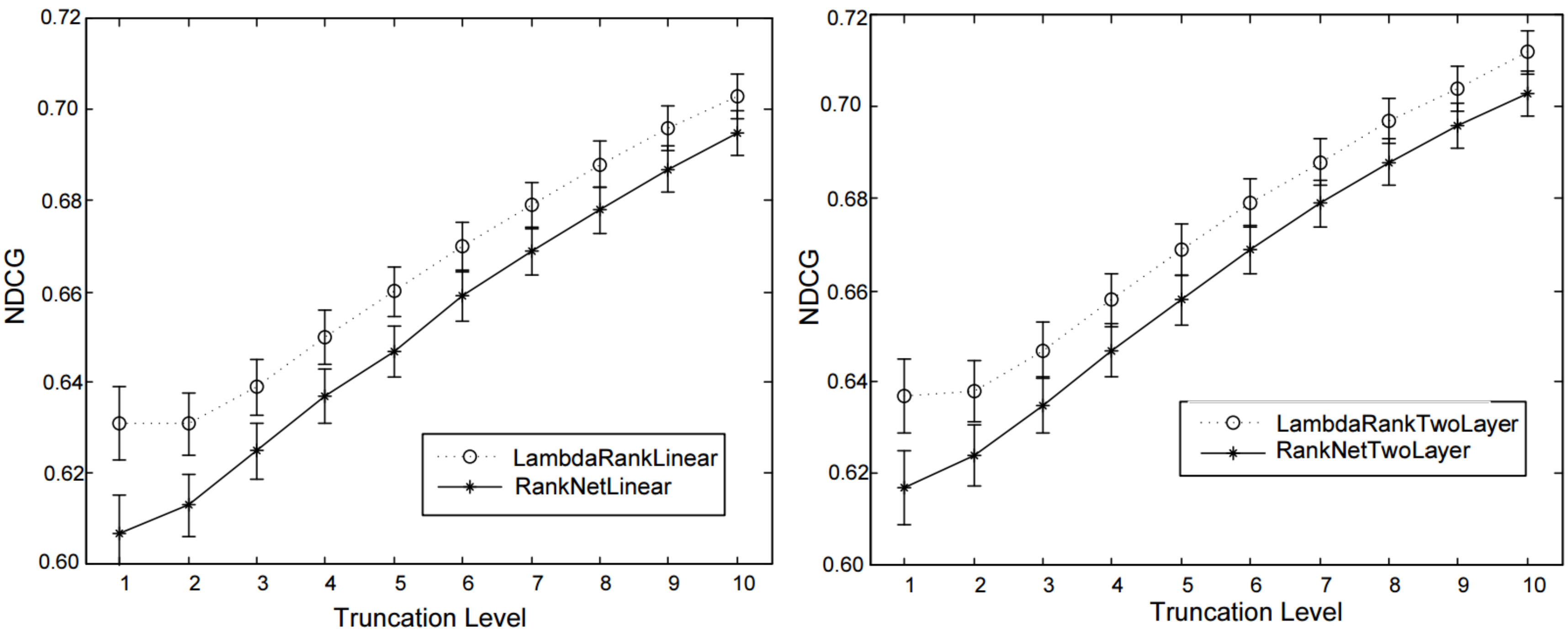
LambdaRank vs. RankNet



Linear nets

Burges, Christopher JC, Robert Ragno, and Quoc Viet Le. "Learning to rank with nonsmooth cost functions." *NIPS*. Vol. 6. 2006.

LambdaRank vs. RankNet



Burges, Christopher JC, Robert Ragno, and Quoc Viet Le. "Learning to rank with nonsmooth cost functions." *NIPS*. Vol. 6. 2006.

Summary of Learning to Rank

- Pointwise, pairwise and listwise approaches for learning to rank
- Pairwise approaches are still the most popular
  - A balance of ranking effectiveness and training efficiency
- LambdaRank is a pairwise approach with list-level information
  - Easy to implement, easy to improve and adjust

A Data Mining Application: Recommendation

Overview

Collaborative Filtering

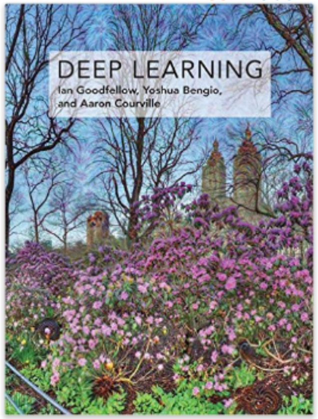
Rating prediction

Top-N ranking

Sincerely thank Prof. Jun Wang

Book Recommendation

Look inside



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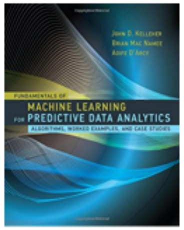

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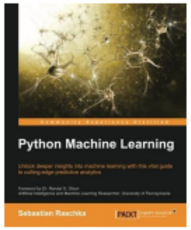

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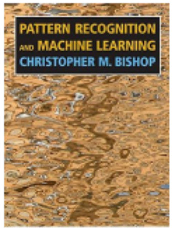

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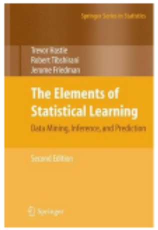

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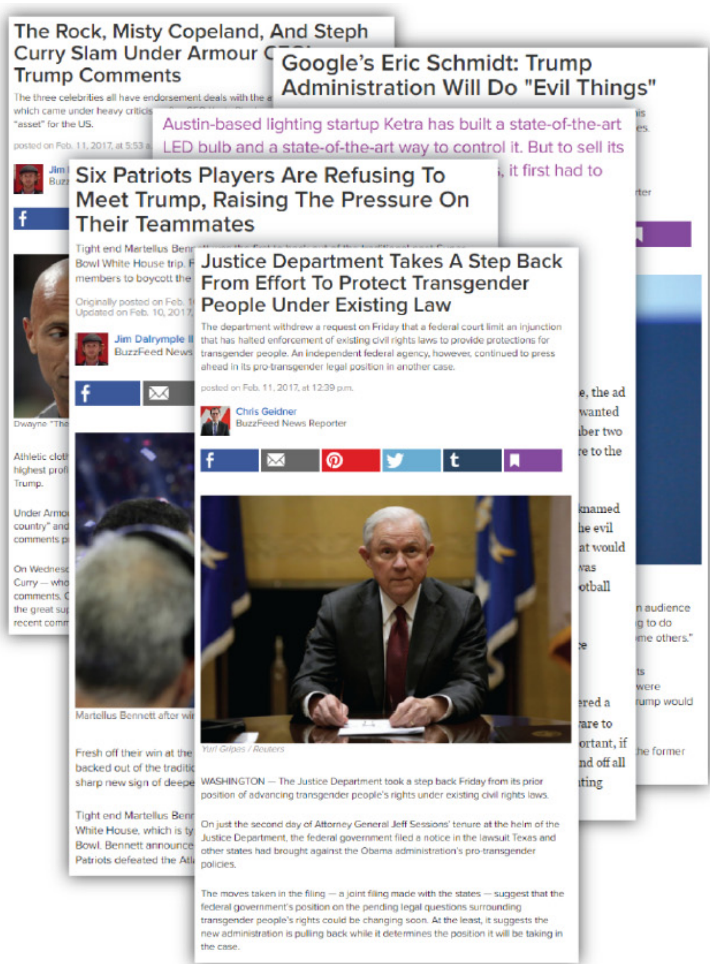
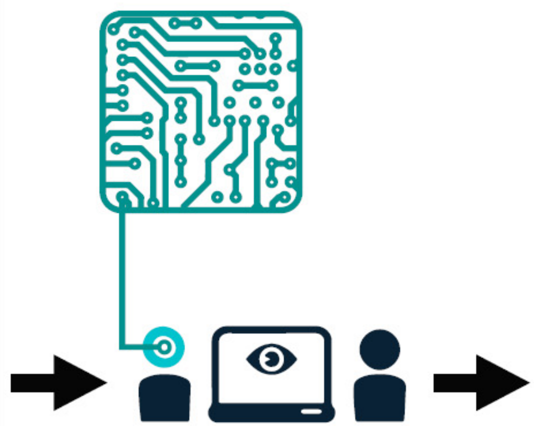
News Feed Recommendation

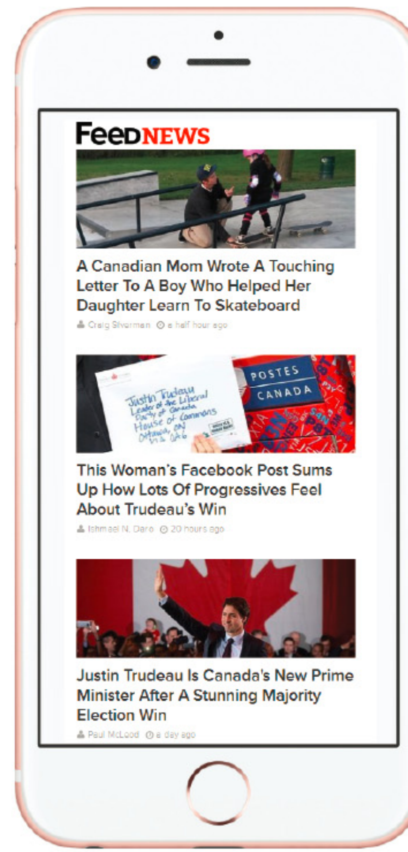


Huge numbers of candidate articles daily

Recommender System



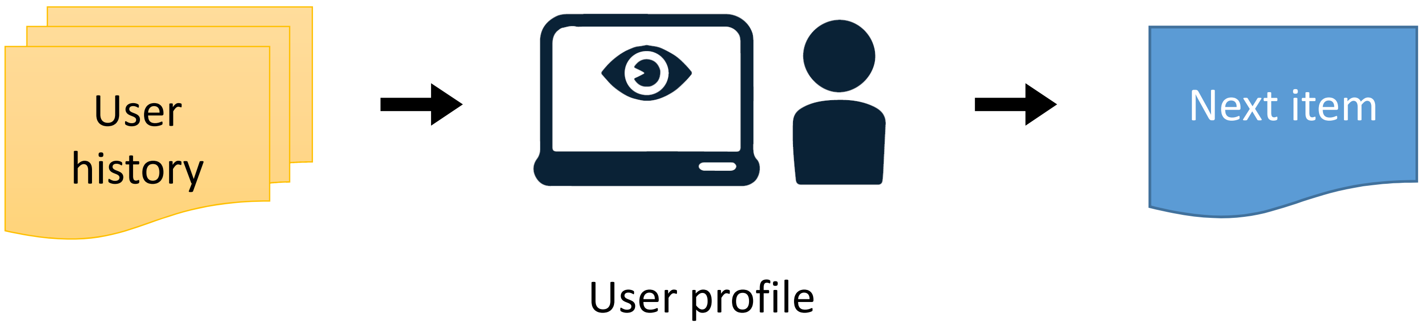
Editors manually select quality articles



Quality articles selected for news feed to end users

Personalized Recommendation

- Personalization framework



Build user profile from her history

- Ratings [amazon.com]
  - Explicit, but time-consuming
- Visits [newsfeed]
  - Implicit

Personalization Methodologies

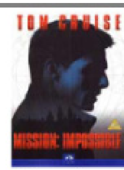
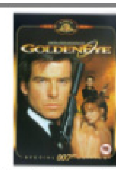
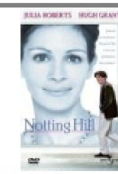
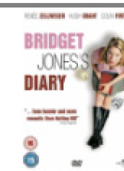
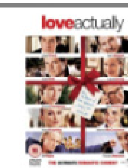
- Given the user's previous liked movies, how to recommend more movies she would like?
  - Method 1: recommend the movies that share the actors/actresses/director/genre with those the user likes
  - Method 2: recommend the movies that the users with similar interest to her like

Information Filtering

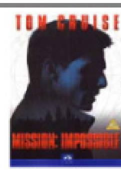
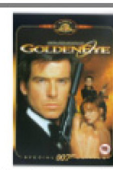
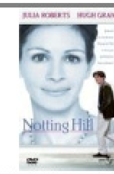
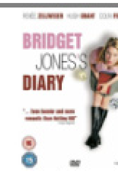
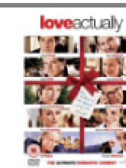
- Information filtering deals with the delivery of information that the user is likely to find interesting or useful
  - Recommender system: information filtering in the form of suggestions
  - Two approaches for information filtering
    - Content-based filtering
      - recommend the movies that share the actors/actresses/director/genre with those the user likes
    - Collaborative filtering (the focus of this lecture)
      - recommend the movies that the users with similar interest to her like

http://recommender-systems.org/information-filtering/

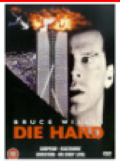
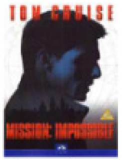
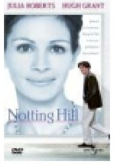
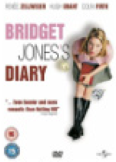
A (small) Rating Matrix

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

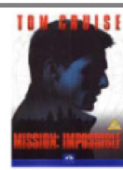
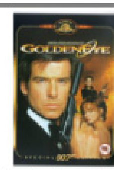
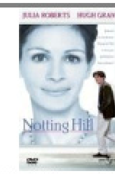
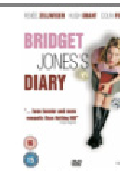
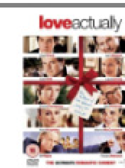
The Users

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

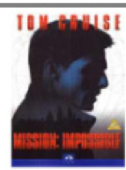
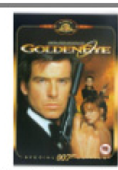
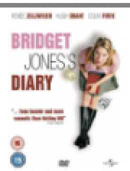
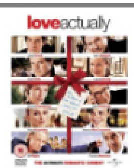
The Items

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

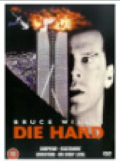
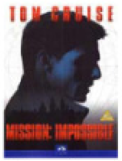
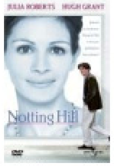
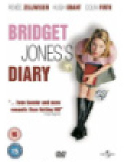
A User-Item Rating

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	$\boxed{★★★★★}$	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

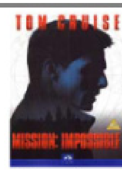
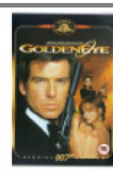
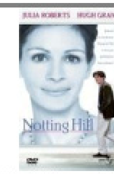
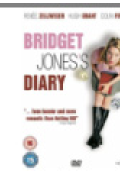
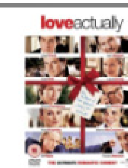
A User Profile

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

An Item Profile

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

A Null Rating Entry

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

- Recommendation on explicit data
  - Predict the null ratings

If I watched *Love Actually*, how would I rate it?

K Nearest Neighbor Algorithm (KNN)

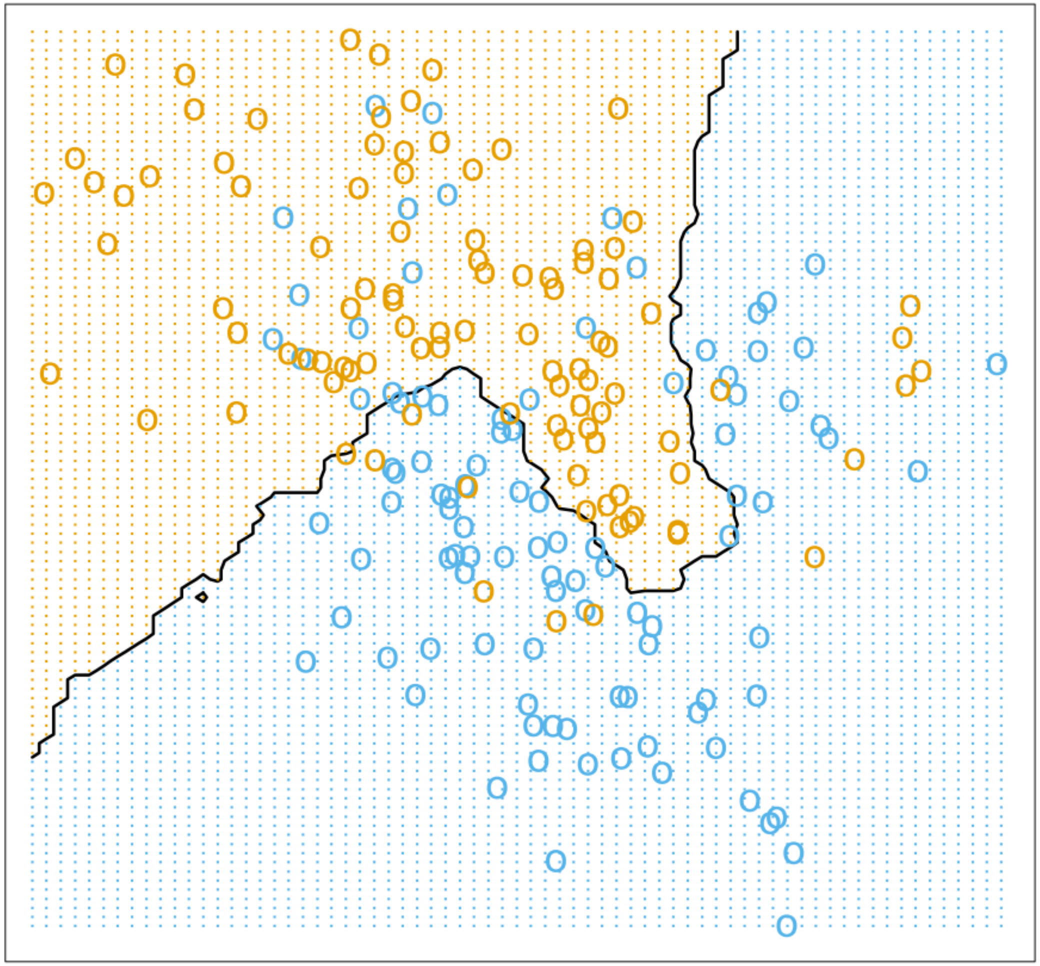
- A non-parametric method used for classification and regression

  - for each input instance *x*, find *k* closest training instances $N_k(x)$ in the feature space
  - the prediction of *x* is based on the average of labels of the *k* instances

$$\hat{y}(x) = \frac{1}{k} \sum_{x_i \in N_k(x)} y_i$$

  - For classification problem, it is the majority voting among neighbors

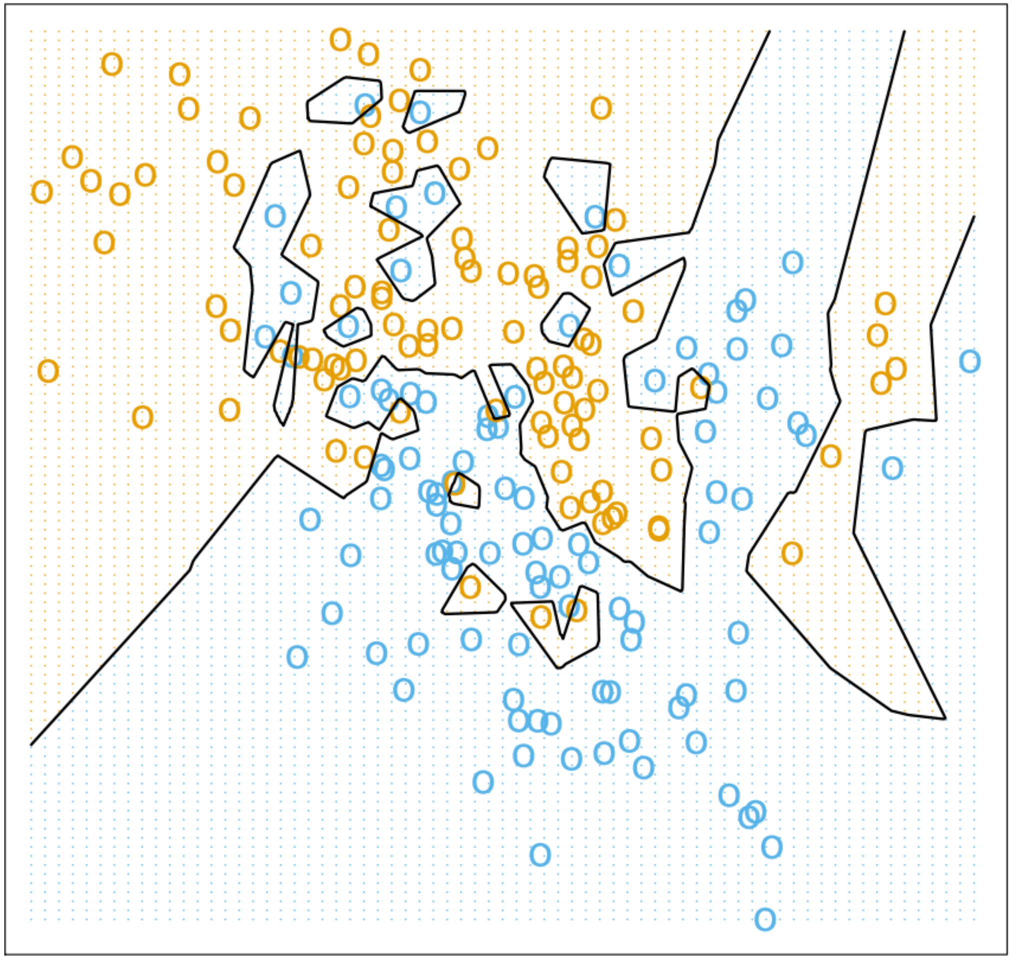
kNN Example



Jerome H. Friedman, Robert Tibshirani, and Trevor Hastie. “The Elements of Statistical Learning”. Springer 2009.

kNN Example

1-nearest neighbor



Jerome H. Friedman, Robert Tibshirani, and Trevor Hastie. “The Elements of Statistical Learning”. Springer 2009.

K Nearest Neighbor Algorithm (KNN)

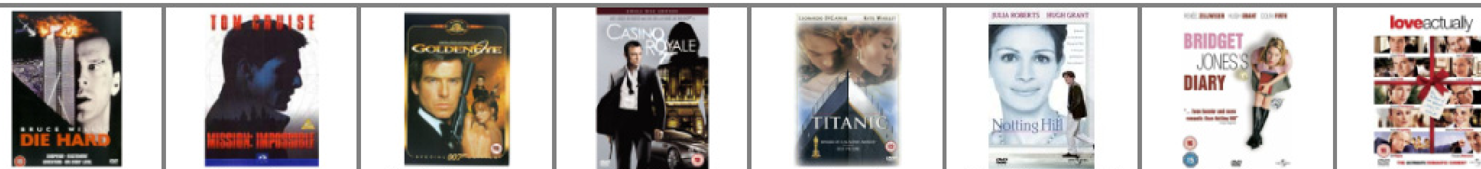
- Generalized version
  - Define similarity function $s(x, x_i)$ between the input instance x and its neighbor x_i
  - Then the prediction is based on the weighted average of the neighbor labels based on the similarities

$$\hat{y}(x) = \frac{\sum_{x_i \in N_k(x)} s(x, x_i) y_i}{\sum_{x_i \in N_k(x)} s(x, x_i)}$$

Non-Parametric kNN

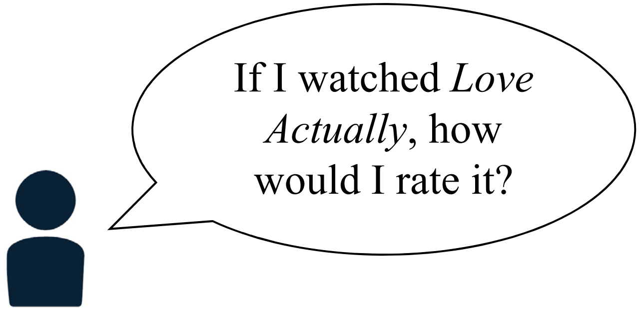
- No parameter to learn
  - In fact, there are *N* parameters: each instance is a parameter
  - There are *N*/*k* effective parameters
    - Intuition: if the neighborhoods are non-overlapping, there would be *N*/*k* neighborhoods, each of which fits one parameter
- Hyperparameter *k*
  - We cannot use sum-of-squared error on the training set as a criterion for picking *k*, since *k*=1 is always the best
  - Tune *k* on validation set

A Null Rating Entry

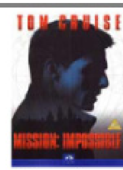
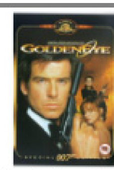
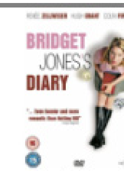
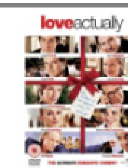


	Die Hard	Mission: Impossible	GoldenEye	Casino Royale	Titanic	Notting Hill	Bridget Jones's Diary	Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

- Recommendation on explicit data
  - Predict the null ratings

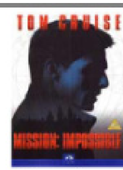
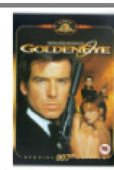
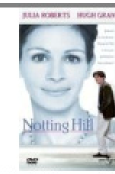
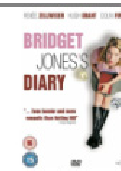
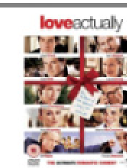


Collaborative Filtering Example

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		**?**
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

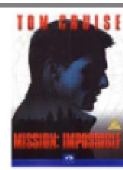
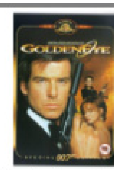
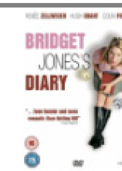
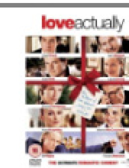
- What do you think the rating would be?

User-based kNN Solution

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		?
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

- Find similar users (neighbors) for Boris
  - Dave and George

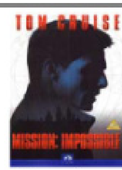
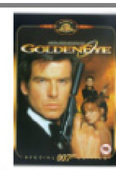
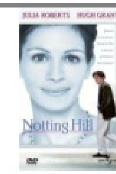
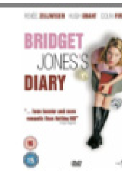
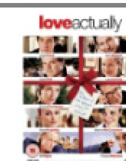
Rating Prediction

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
▶ **Boris**	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		★½☆☆☆ ◀
▶ **Dave**		★★★★★	★★★★★	★★★★★				★☆☆☆☆ ◀
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
▶ **George**	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆ ◀

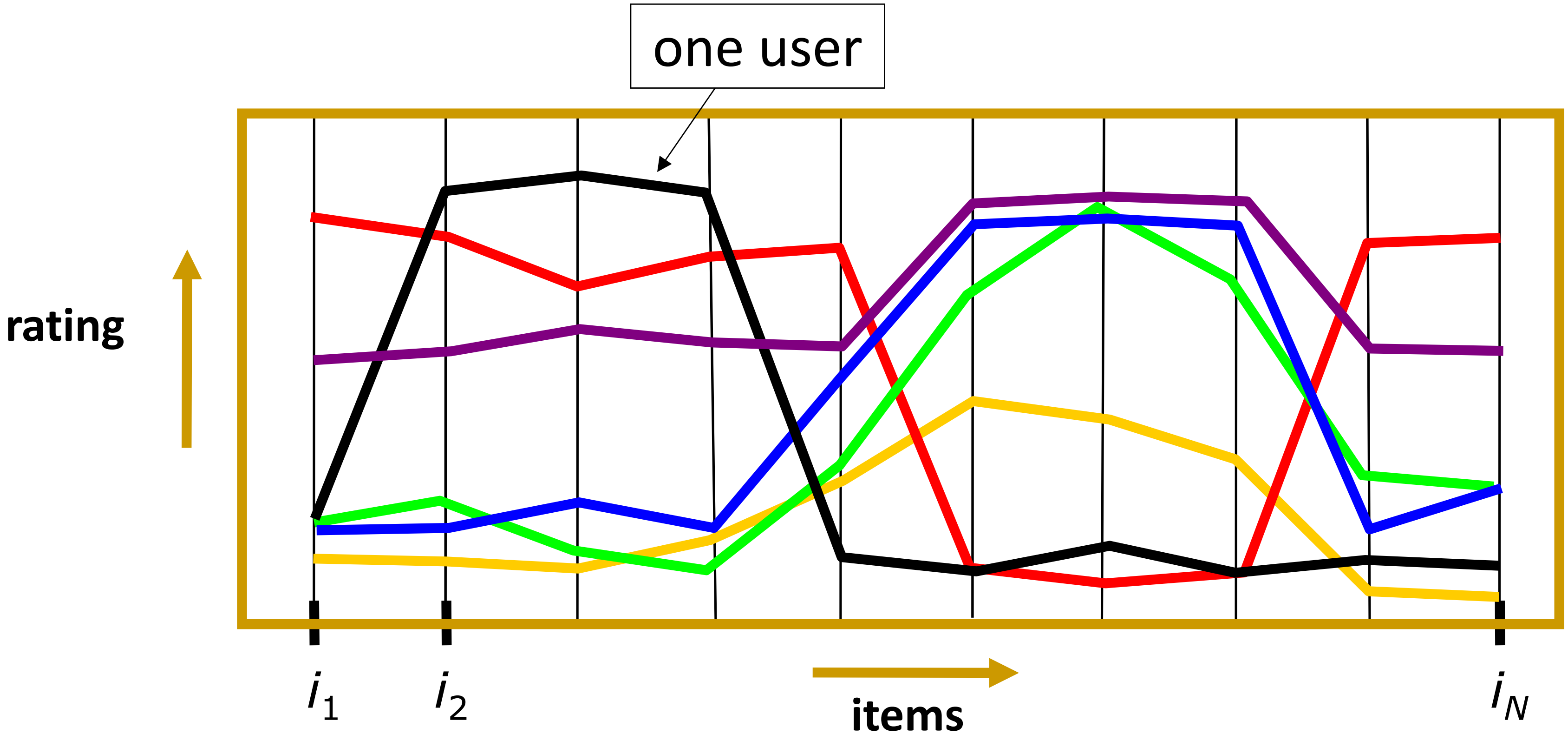
- Average Dave's and George's rating on Love Actually
  - Prediction = (1+2)/2 = 1.5

Collaborative Filtering for Recommendation

- Basic user-based kNN algorithm
  - For each target user for recommendation
    1. Find similar users
    2. Based on similar users, recommend new items

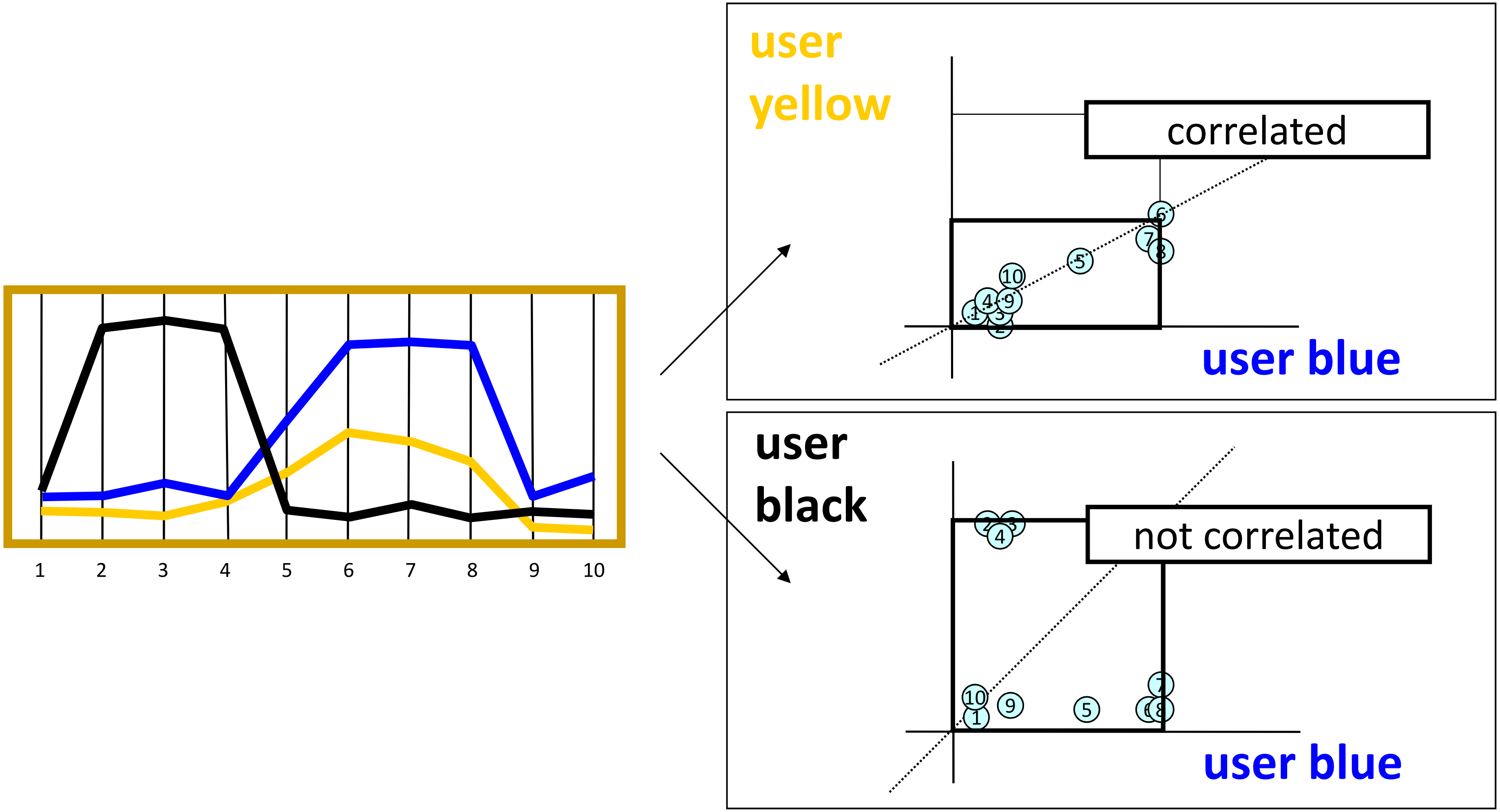
	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		★★☆☆☆
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

Similarity between Users

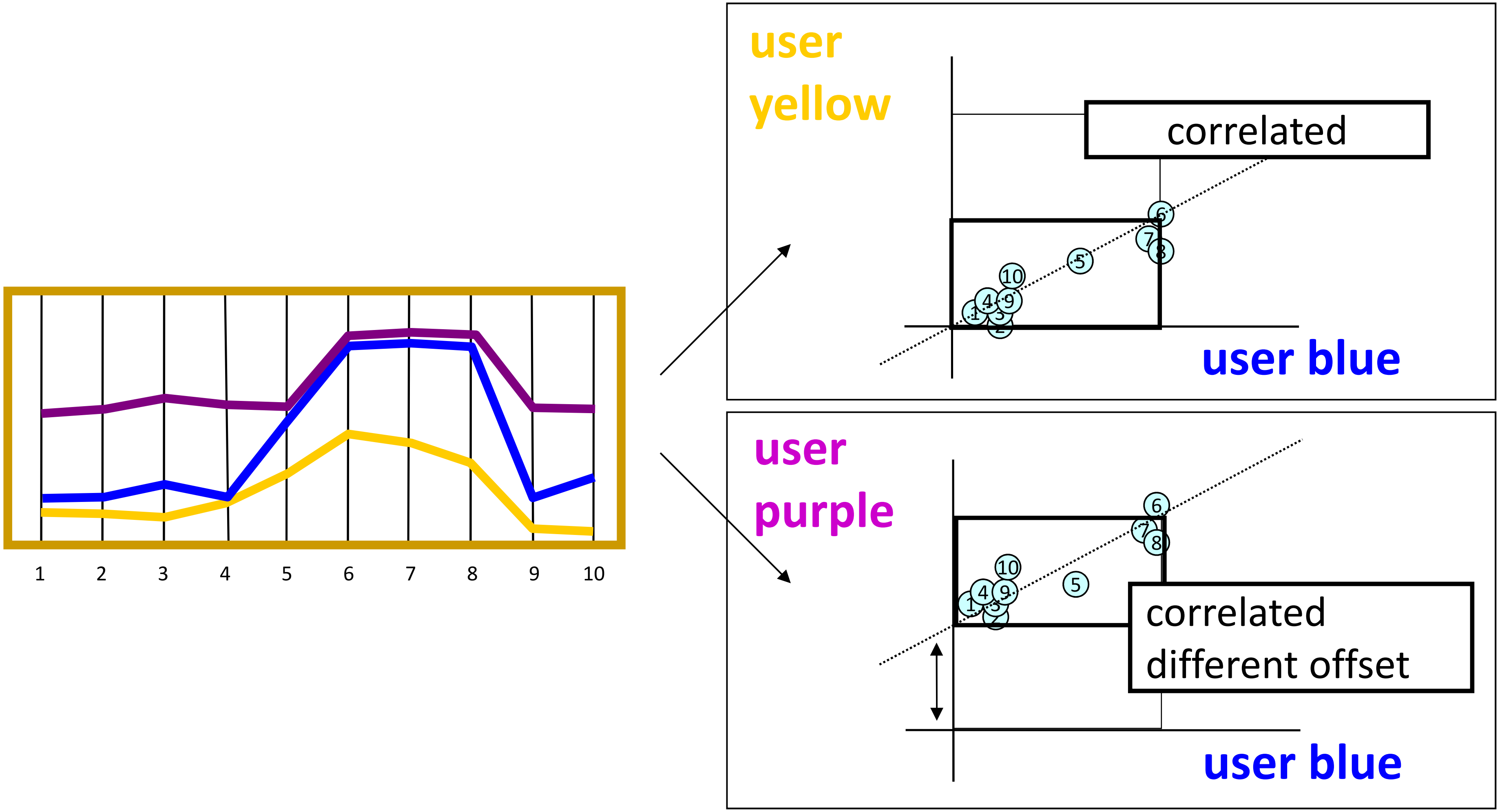


- Each user's profile can be directly built as a vector based on her item ratings

Similarity between Users



Similarity between Users



Similarity Measures (Users)

- Similarity measures between two users *a* and *b*
  - Cosine (angle)

$$s_u^{\cos}(u_a, u_b) = \frac{u_a^\top u_b}{\|u_a\|\|u_b\|} = \frac{\sum_m x_{a,m} x_{b,m}}{\sqrt{\sum_m x_{a,m}^2 \sum_m x_{b,m}^2}}$$

  - Pearson Correlation

$$s_u^{\mathrm{corr}}(u_a, u_b) = \frac{\sum_m (x_{a,m} - \bar{x}_a)(x_{b,m} - \bar{x}_b)}{\sqrt{\sum_m (x_{a,m} - \bar{x}_a)^2 \sum_m (x_{b,m} - \bar{x}_b)^2}}$$

User-based kNN Rating Prediction

- Predicting the rating from target user *t* to item *m*

Weights according to similarity with query user

predict according to rating of similar user (corrected for average rating of that user)

$$\hat{x}_{t,m} = \bar{x}_t + \frac{\sum_{u_a \in N_u(u_t)} s_u(u_t, u_a)(x_{a,m} - \bar{x}_a)}{\sum_{u_a \in N_u(u_t)} s_u(u_t, u_a)}$$

correct for average rating query user

$$\bar{u}_t = \frac{1}{|I(u_t)|} \sum_{m \in I(u_t)} x_{t,m}$$

normalize such that ratings stay between 0 and *R*

Item-based kNN Solution

- Recommendation based on item similarity

query item

similar items based on profiles

	i_1	i_2	i_3	i_4	i_5	…	i_M
u_1	2	0	3	2	5	…	1
u_2	0	4	0	0	0	…	5
u_3	0	2	0	0	0	…	4
u_4	1	0	4	2	4	…	2
…	…	…	…	…	…	…	…
u_K	2	0	4	?	4	…	1

target user

predicted rating: 4

Item-based kNN Solution

- For each unrated items *m* of the target user *t*
  - Find similar items {*a*}
  - Based on the set of similar items {*a*}
    - Predict the rating of the item *m*

query item

similar items based on profiles

	i_1	i_2	i_3	i_4	i_5	…	i_M
u_1	2	0	3	2	5	…	1
u_2	0	4	0	0	0	…	5
u_3	0	2	0	0	0	…	4
u_4	1	0	4	2	4	…	2
…	…	…	…	…	…	…	…
u_K	2	0	$\boxed{4}$	?	$\boxed{4}$	…	1

target user

predicted rating: 4

Similarity Measures (Items)

- Similarity measures between two items *a* and *b*
  - Cosine (angle)

$$s_i^{\cos}(i_a, i_b) = \frac{i_a^\top i_b}{\|i_a\| \|i_b\|} = \frac{\sum_u x_{u,a} x_{u,b}}{\sqrt{\sum_u x_{u,a}^2 \sum_u x_{u,b}^2}}$$

  - Adjusted Cosine

$$s_i^{\mathrm{adcos}}(i_a, i_b) = \frac{\sum_u (x_{u,a} - \bar{x}_u)(x_{u,b} - \bar{x}_u)}{\sqrt{\sum_u (x_{u,a} - \bar{x}_u)^2 \sum_u (x_{u,b} - \bar{x}_u)^2}}$$

  - Pearson Correlation

$$s_i^{\mathrm{corr}}(i_a, i_b) = \frac{\sum_u (x_{u,a} - \bar{x}_a)(x_{u,b} - \bar{x}_b)}{\sqrt{\sum_u (x_{u,a} - \bar{x}_a)^2 \sum_u (x_{u,b} - \bar{x}_b)^2}}$$

Item-based kNN Rating Prediction

- Get top-*k* neighbor items that the target user *t* rated

Rank position

$$N_i(u_t, i_a) = \{i_b | r_i(i_a, i_b) < K^*, x_{t,b} \neq 0\}$$

Choose *K** such that $|N_i(u_t, i_a)| = k$

- Predict ratings for item *a* that the target user *t* did not rate

$$\hat{x}_{t,a} = \frac{\sum_{i_b \in N_i(u_t, i_a)} s_i(i_a, i_b) x_{t,b}}{\sum_{i_b \in N_i(u_t, i_a)} s_i(i_a, i_b)}$$

Don't need to correct for users average rating since query user itself is used to do predictions

Empirical Study

- Movielens dataset from grouplens
    - http://www.grouplens.org/node/73
    - Users visit Movielens
        - rate and receive recommendations for movies
    - Dataset (ML-100k)
        - 100k ratings from 1 to 5
        - 943 users, 1682 movies (rated by at least one user)
        - Sparsity level

$$1 - \frac{\#\text{non-zero entries}}{\text{total entries}} = 1 - \frac{10^5}{943 \times 1682} = 93.69\%$$

Experiment Setup

- Split data in training (x%) and test set ((100-x)%)
  - Can be repeated T times and results averaged

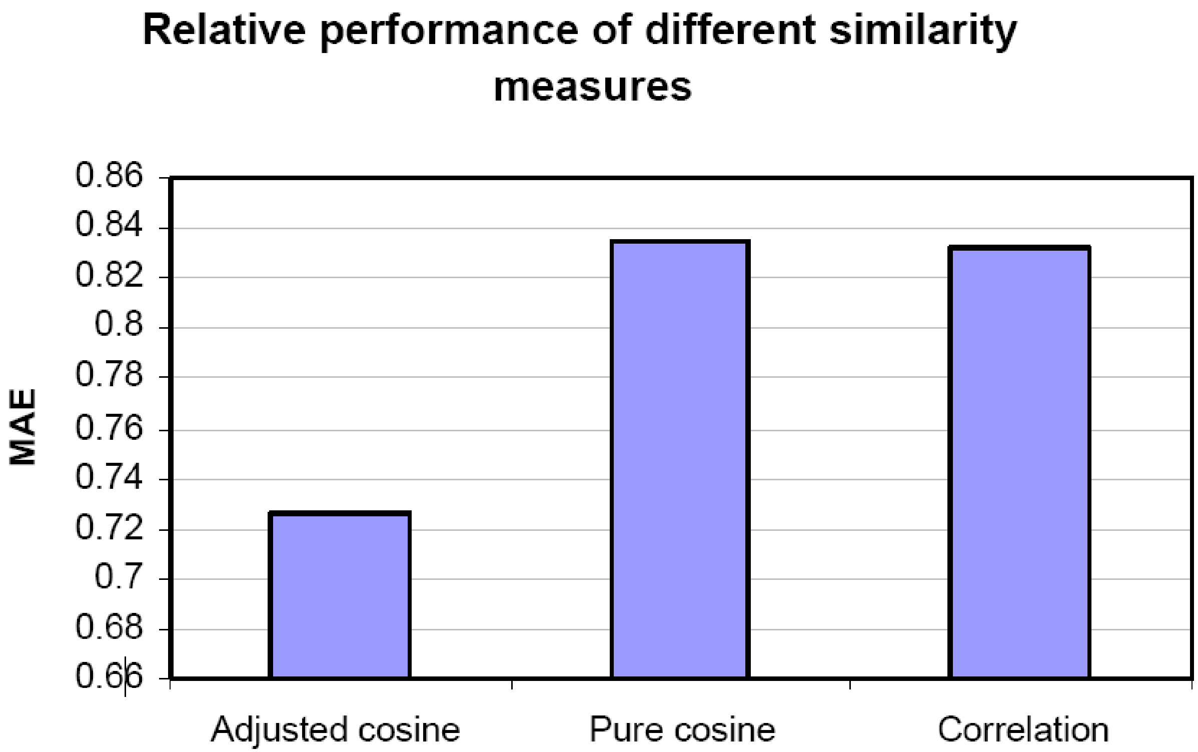
- Evaluation metrics
  - Mean-Absolute Error (MAE)

$$\mathrm{MAE} = \frac{1}{|D_{\mathrm{test}}|} \sum_{(u,i,r) \in D_{\mathrm{test}}} |r - \hat{r}_{u,i}|$$

  - Root Mean Squared Error (RMSE)

$$\mathrm{RMSE} = \sqrt{\frac{1}{|D_{\mathrm{test}}|} \sum_{(u,i,r) \in D_{\mathrm{test}}} (r - \hat{r}_{u,i})^2}$$

Impact of Similarity Measures

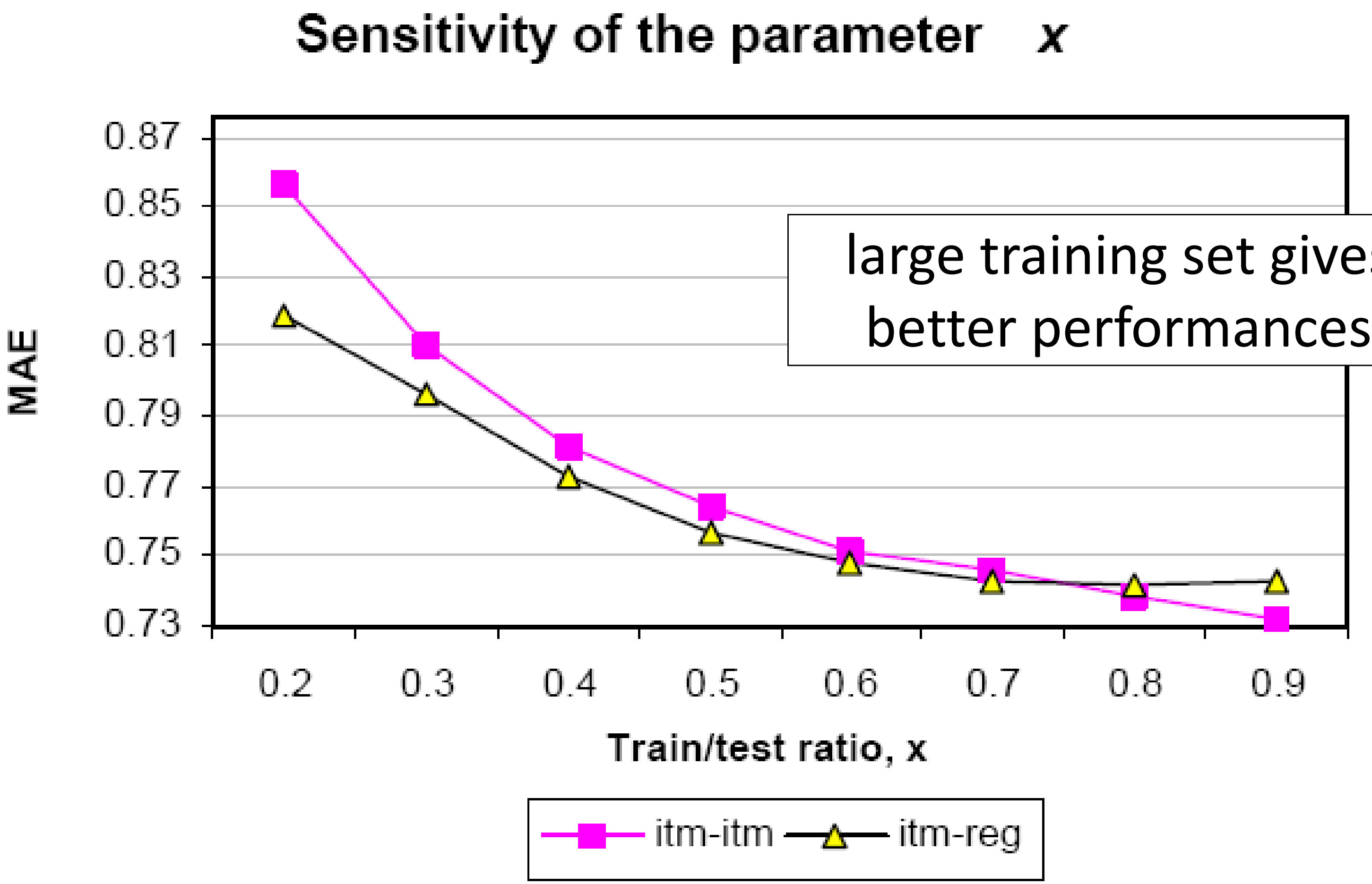


$$s_i^{\mathrm{adcos}}(i_a, i_b) = \frac{\sum_u (x_{u,a} - \bar{x}_u)(x_{u,b} - \bar{x}_u)}{\sqrt{\sum_u (x_{u,a} - \bar{x}_u)^2 \sum_u (x_{u,b} - \bar{x}_u)^2}}$$

$$s_i^{\mathrm{corr}}(i_a, i_b) = \frac{\sum_u (x_{u,a} - \bar{x}_a)(x_{u,b} - \bar{x}_b)}{\sqrt{\sum_u (x_{u,a} - \bar{x}_a)^2 \sum_u (x_{u,b} - \bar{x}_b)^2}}$$

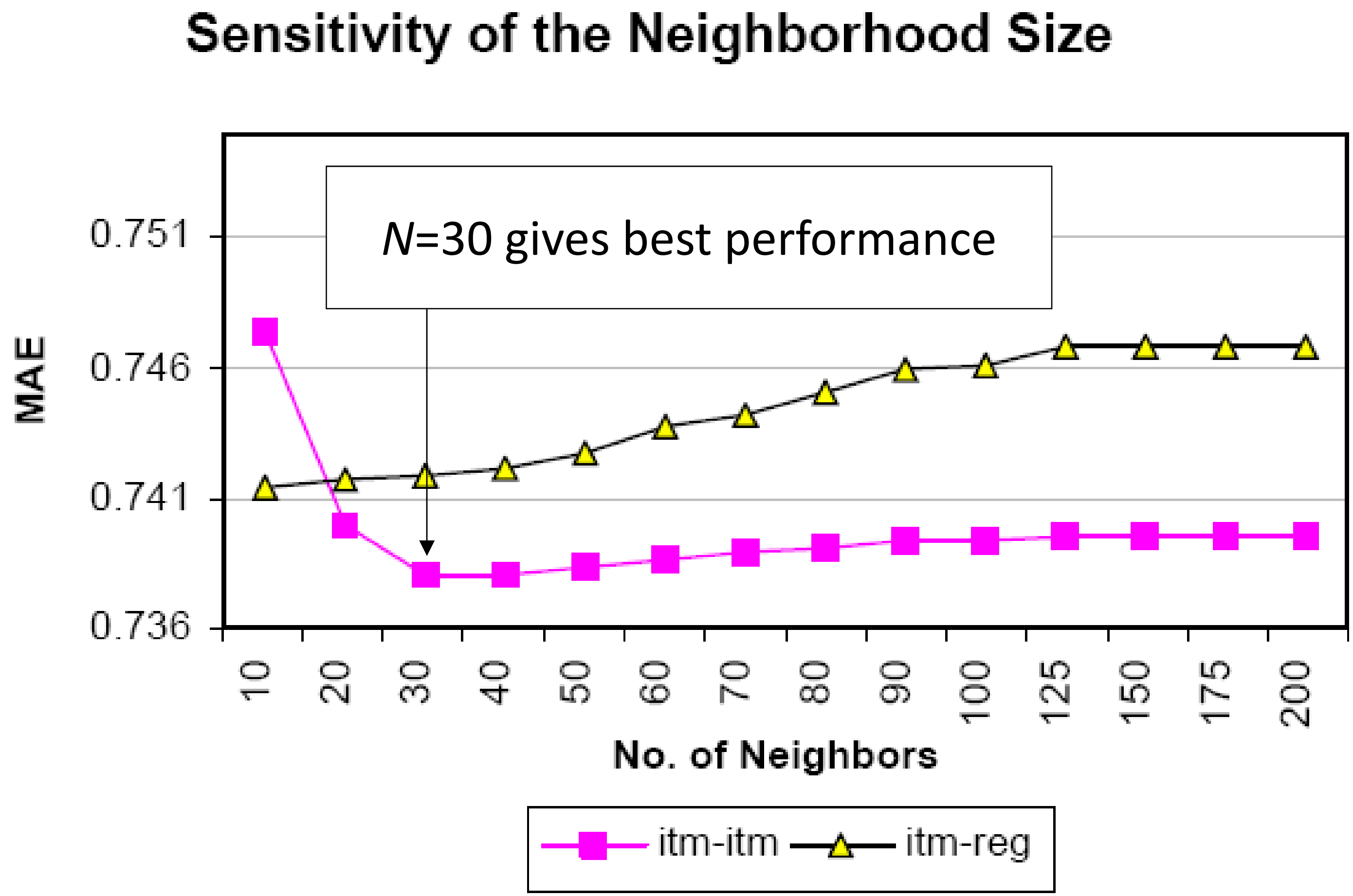
$$s_i^{\cos}(i_a, i_b) = \frac{i_a^\top i_b}{\|i_a\| \|i_b\|} = \frac{\sum_u x_{u,a} x_{u,b}}{\sqrt{\sum_u x_{u,a}^2 \sum_u x_{u,b}^2}}$$

Sensitivity of Train/Test Ratio

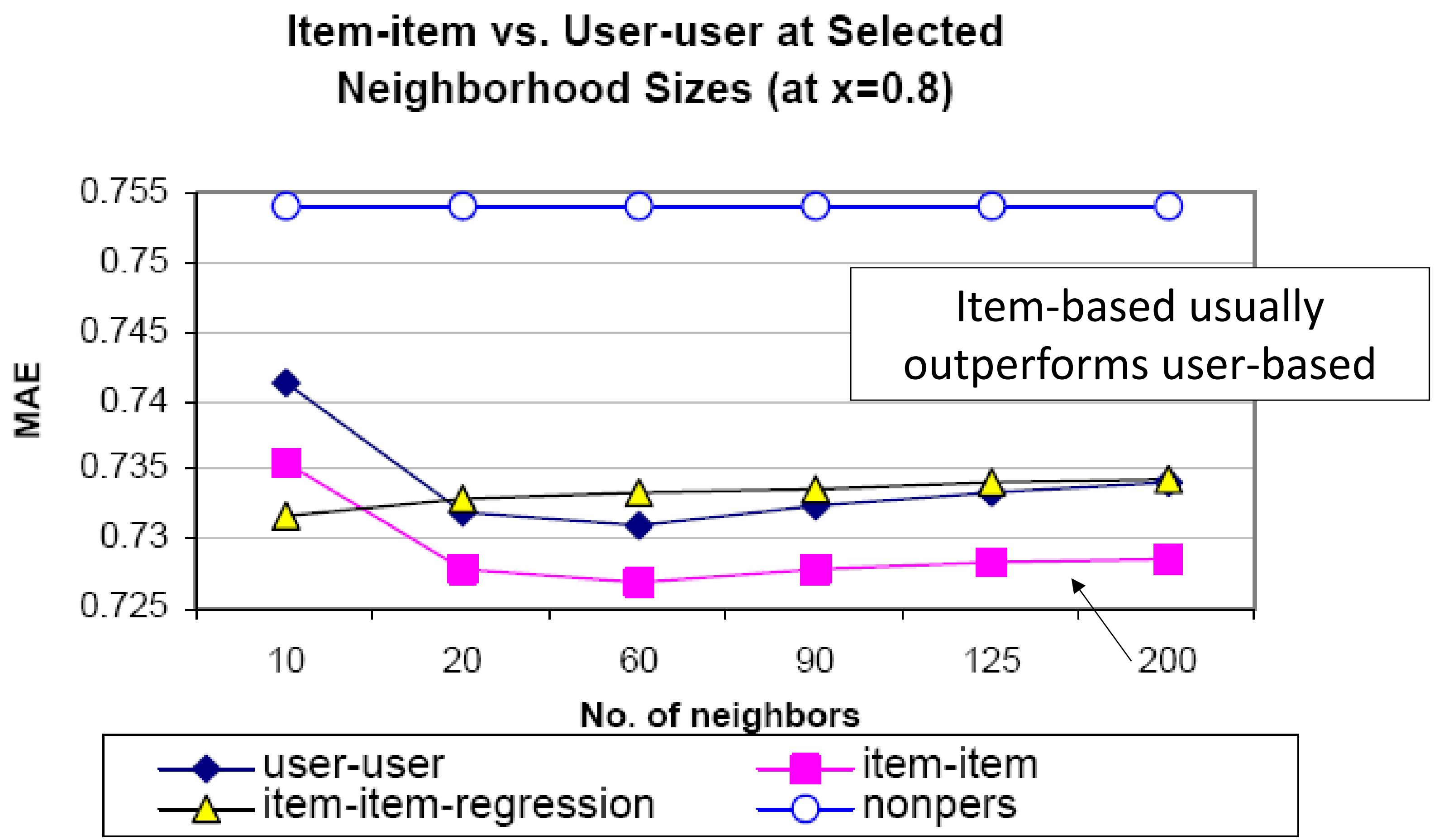


large training set gives better performances

Sensitivity Neighborhood Size *k*



Item-based vs. User-based



- Item-item similarity is usually more stable and objective

kNN based Methods Summary

- Straightforward and highly explainable
- No parameter learning
  - Only one hyperparameter *k* to tune
  - Cannot get improved by learning
- Efficiency could be a serious problem
  - When the user/item numbers are large
  - When there are a huge number of user-item ratings
- We may need a parametric and learnable model

Matrix Factorization Techniques

	Die Hard	Mission: Impossible	GoldenEye	Casino Royale	Titanic	Notting Hill	Bridget Jones's Diary	Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		★★☆☆☆
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

Koren, Yehuda, Robert Bell, and Chris Volinsky. "Matrix factorization techniques for recommender systems." *Computer* 42.8 (2009).

Matrix Factorization Techniques

Items

Users

1		3			5			5		4	
		5	4	?		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

$$\hat{r}_{u,i} = p_u^\top q_i$$

$\simeq$

Users

	.1	-.4	.2
u	-.5	.6	.5
	-.2	.3	.5
	1.1	2.1	.3
	-.7	2.1	-2
	-1	.7	.3

$\cdot$

Items

				i							
1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

Basic MF Model

- Prediction of user *u*'s rating on item *i*

$$\hat{r}_{u,i} = p_u^\top q_i$$

Bilinear model

- Loss function

$$\mathcal{L}(u, i, r_{u,i}) = \frac{1}{2}(r_{u,i} - p_u^\top q_i)^2$$

- Training objective

$$\min_{P,Q} \sum_{r_{u,i} \in D} \frac{1}{2}(r_{u,i} - p_u^\top q_i)^2 + \frac{\lambda}{2}(\|p_u\|^2 + \|q_i\|^2)$$

- Gradients

$$\frac{\partial \mathcal{L}(u, i, r_{u,i})}{\partial p_u} = (p_u^\top q_i - r_{u,i})q_i + \lambda p_u$$

$$\frac{\partial \mathcal{L}(u, i, r_{u,i})}{\partial q_i} = (p_u^\top q_i - r_{u,i})p_u + \lambda q_i$$

MF with Biases

- Prediction of user *u*'s rating on item *i*

$$\hat{r}_{u,i} = \underbrace{\mu}_{\text{Global bias}} + \underbrace{b_u}_{\text{User bias}} + \underbrace{b_i}_{\text{Item bias}} + \underbrace{p_u^\top q_i}_{\text{User-item Interaction}}$$

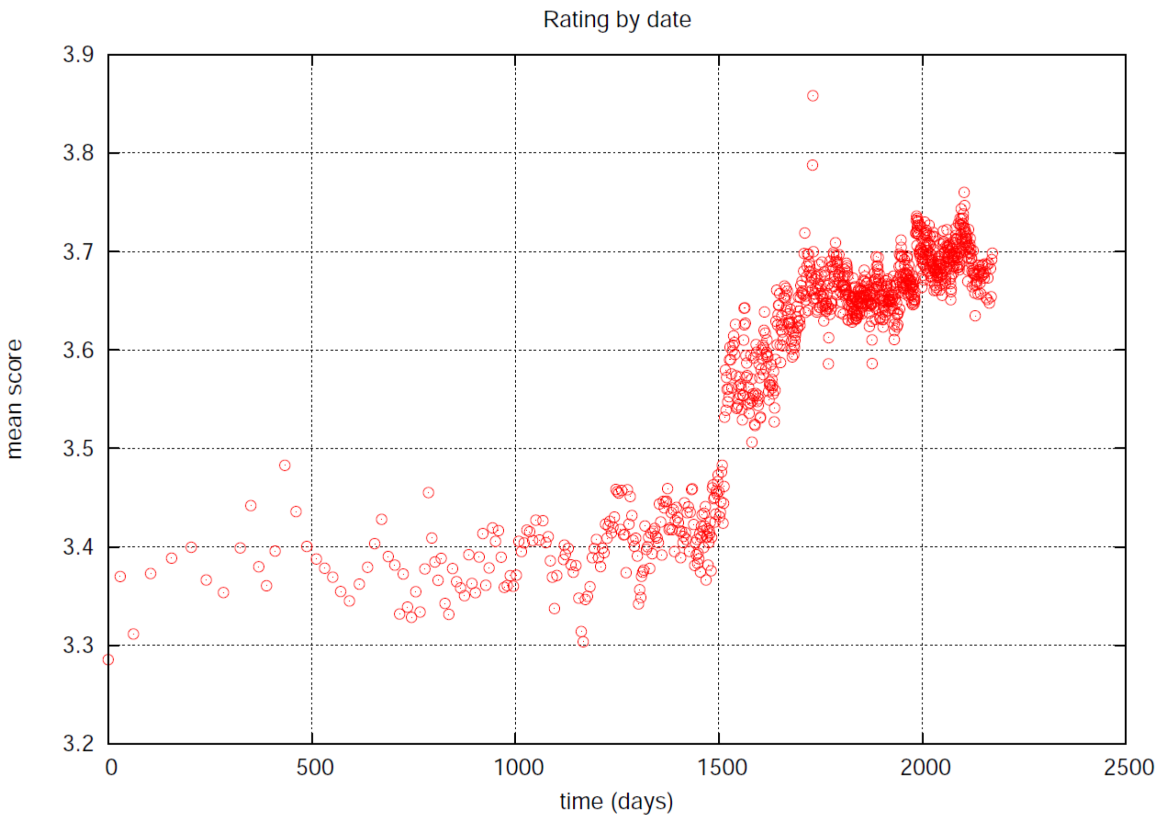
- Training objective

$$\min_{P,Q} \sum_{r_{u,i} \in D} \frac{1}{2}\Big(r_{u,i} - (\mu + b_u + b_i + p_u^\top q_i)\Big)^2 + \frac{\lambda}{2}(\|p_u\|^2 + \|q_i\|^2 + b_u^2 + b_i^2)$$

- Gradient update

$$\begin{aligned}
\delta &= r_{u,i} - (\mu + b_u + b_i + p_u^\top q_i) \\
\mu &\leftarrow \mu + \eta\delta \\
b_u &\leftarrow (1 - \eta\lambda) b_u + \eta\delta \\
b_i &\leftarrow (1 - \eta\lambda) b_i + \eta\delta \\
p_u &\leftarrow (1 - \eta\lambda) p_u + \eta\delta q_i \\
q_i &\leftarrow (1 - \eta\lambda) q_i + \eta\delta p_u
\end{aligned}$$

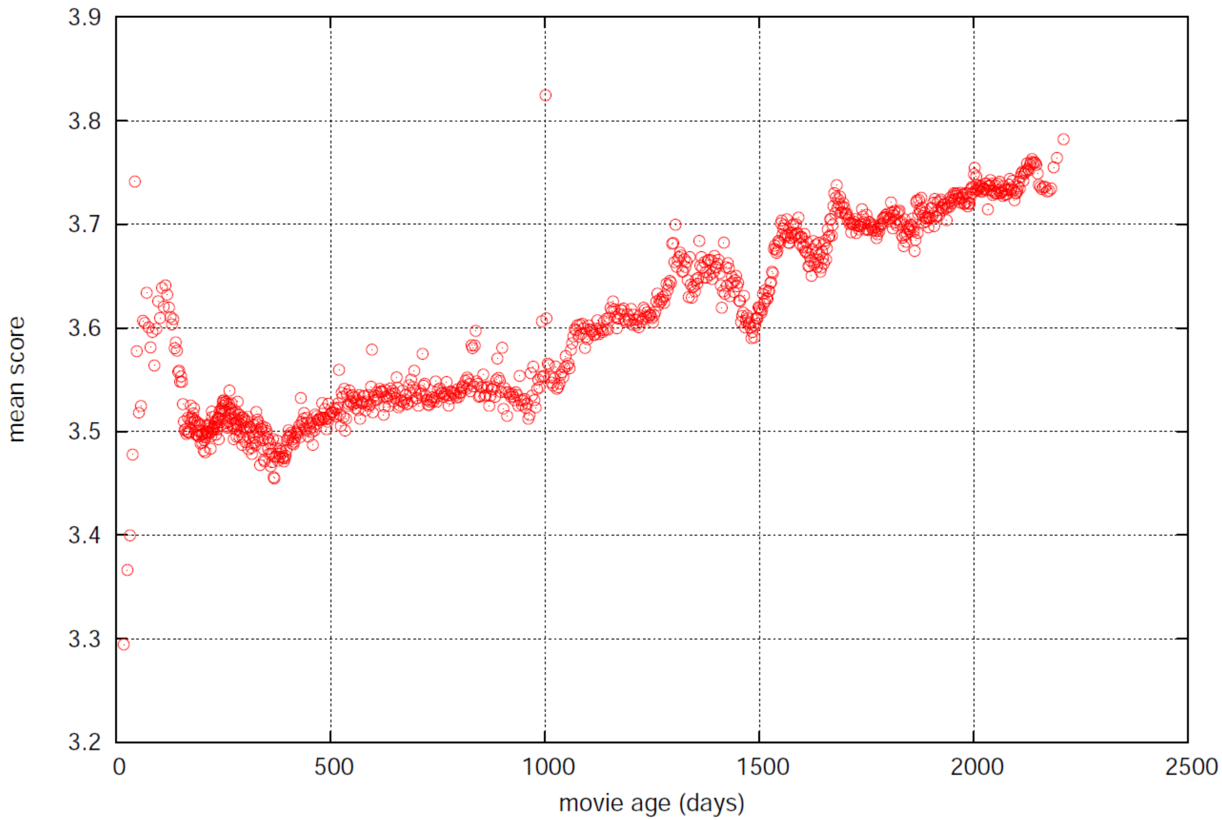
Temporal Dynamics



- A sudden rise in the average movie rating begging around 1500 days (early 2004) into the dataset

Koren, Yehuda. "Collaborative filtering with temporal dynamics." *KDD 2009*.

Temporal Dynamics



- People tend to give higher ratings as movies become older

Koren, Yehuda. "Collaborative filtering with temporal dynamics.“ *KDD 2009*.

Multiple sources of temporal dynamics

- Item-side effects
  - Product perception and popularity are constantly changing
  - Seasonal patterns influence items’ popularity

- User-side effects
  - Customers ever redefine their taste
  - Transient, short-term bias
  - Drifting rating scale
  - Change of rater within household

Addressing temporal dynamics

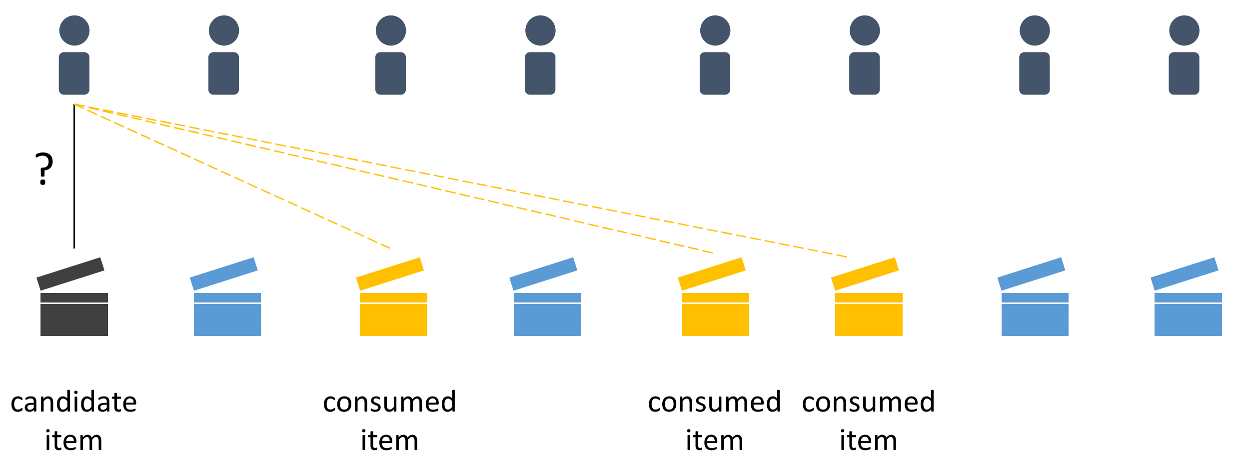
- Factor model conveniently allows separately treating different aspects
- We observe changes in:
  - Rating scale of individual users $b_u(t)$
  - Popularity of individual items $b_i(t)$
  - User preferences $p_u(t)$

$$r_{u,i}(t) = \mu + b_u(t) + b_i(t) + p_u(t)^\top q_i$$

- Design guidelines
  - Items show slower temporal changes
  - Users exhibit frequent and sudden changes
  - Factors $p_u(t)$ are expensive to model
  - Gain flexibility by heavily parameterizing the functions

Neighborhood (Similarity)-based MF

- Assumption: user's previous consumed items reflect her taste
- Derive unknown ratings from those of "similar" items (item-item variant)



Neighborhood based MF modeling: SVD++

$$\hat{r}_{u,i} = b_{u,i} + q_i^\top \left(p_u + |N(u)|^{-\frac{1}{2}} \sum_{j \in N(u)} y_j \right)$$

need to estimate rating of user u for item i

bias

item vector

user vector

Set of user previously rated items

item vector for neighborhood similarity

- Each item has two latent vectors
  - The standard item vector q_i
  - The vector y_i when it is used for estimation the similarity between the candidate item and the target user

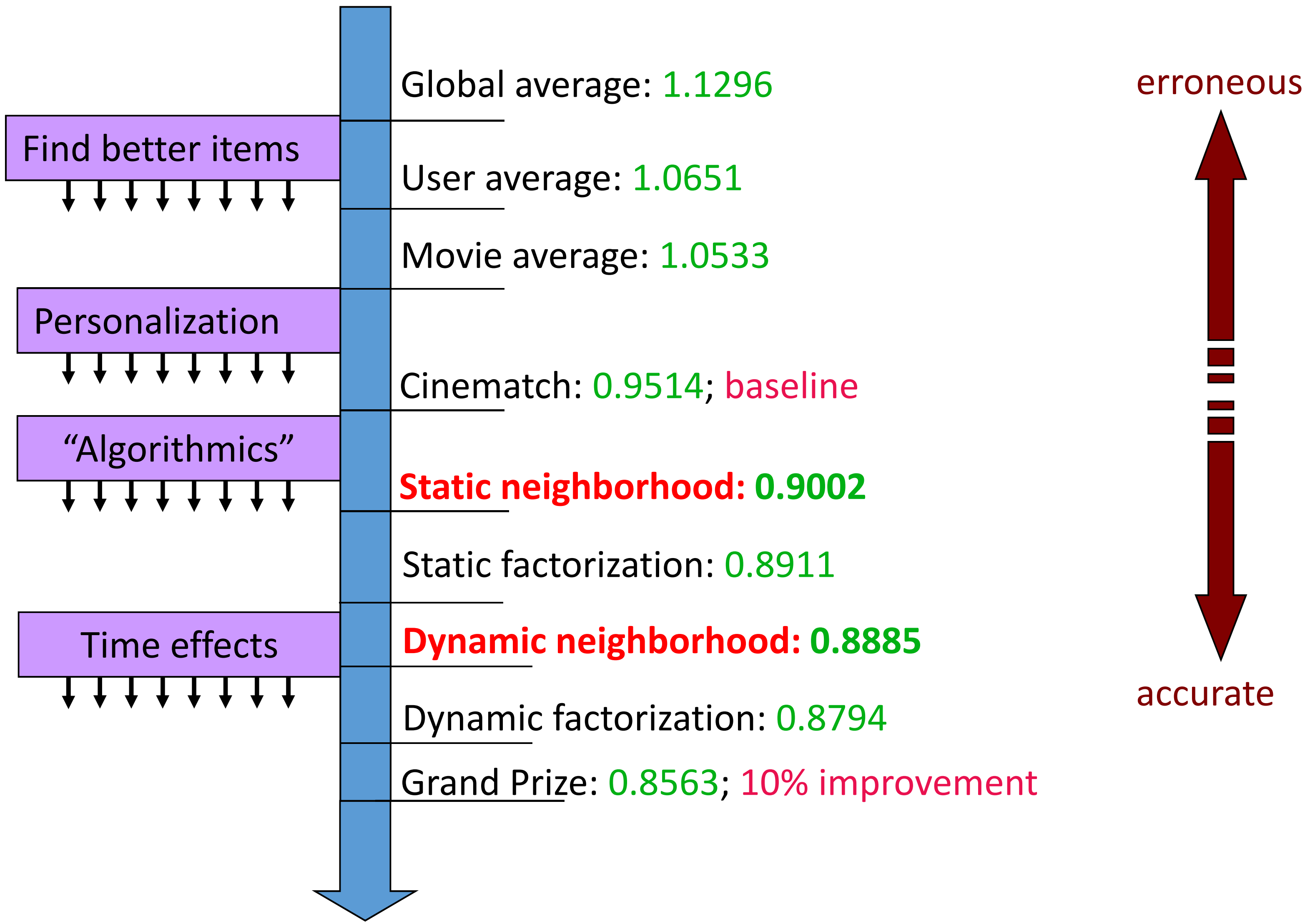
Koren, Yehuda. "Factorization meets the neighborhood: a multifaceted collaborative filtering model." *KDD*, 2008.

Netflix Prize

- An open competition for the best collaborative filtering algorithm for movies
  - Began on October 2, 2006.
  - A million-dollar challenge to improve the accuracy (RMSE) of the Netflix recommendation algorithm by 10%

- Netflix provided
  - Training data: 100,480,507 ratings:
  - 480,189 users x 17,770 movies.
  - Format: <user, movie, date, rating>

- Two popular approaches:
  - Matrix factorization
  - Neighborhood



Temporal neighborhood model delivers same relative RMSE improvement (0.0117) as temporal factor model (!)

Feature-based Matrix Factorization

$$\hat{y} = \mu + \Big(\sum_j b_j^{(g)}\gamma_j + \sum_j b_j^{(u)}\alpha_j + \sum_j b_j^{(i)}\beta_j\Big) + \Big(\sum_j p_j\alpha_j\Big)^\top\Big(\sum_j q_j\beta_j\Big)$$

- Regard all information as features
  - User id and item id
  - Time, item category, user demographics etc.
- User and item features are with latent factors

T. Chen et al. Feature-based matrix factorization. arXiv:1109.2271
http://svdfeature.apexlab.org/wiki/images/7/76/APEX-TR-2011-07-11.pdf

Open source: http://svdfeature.apexlab.org/wiki/Main_Page

Factorization Machine

$$\hat{y}(\mathbf{x}) = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \langle v_i, v_j \rangle x_i x_j$$

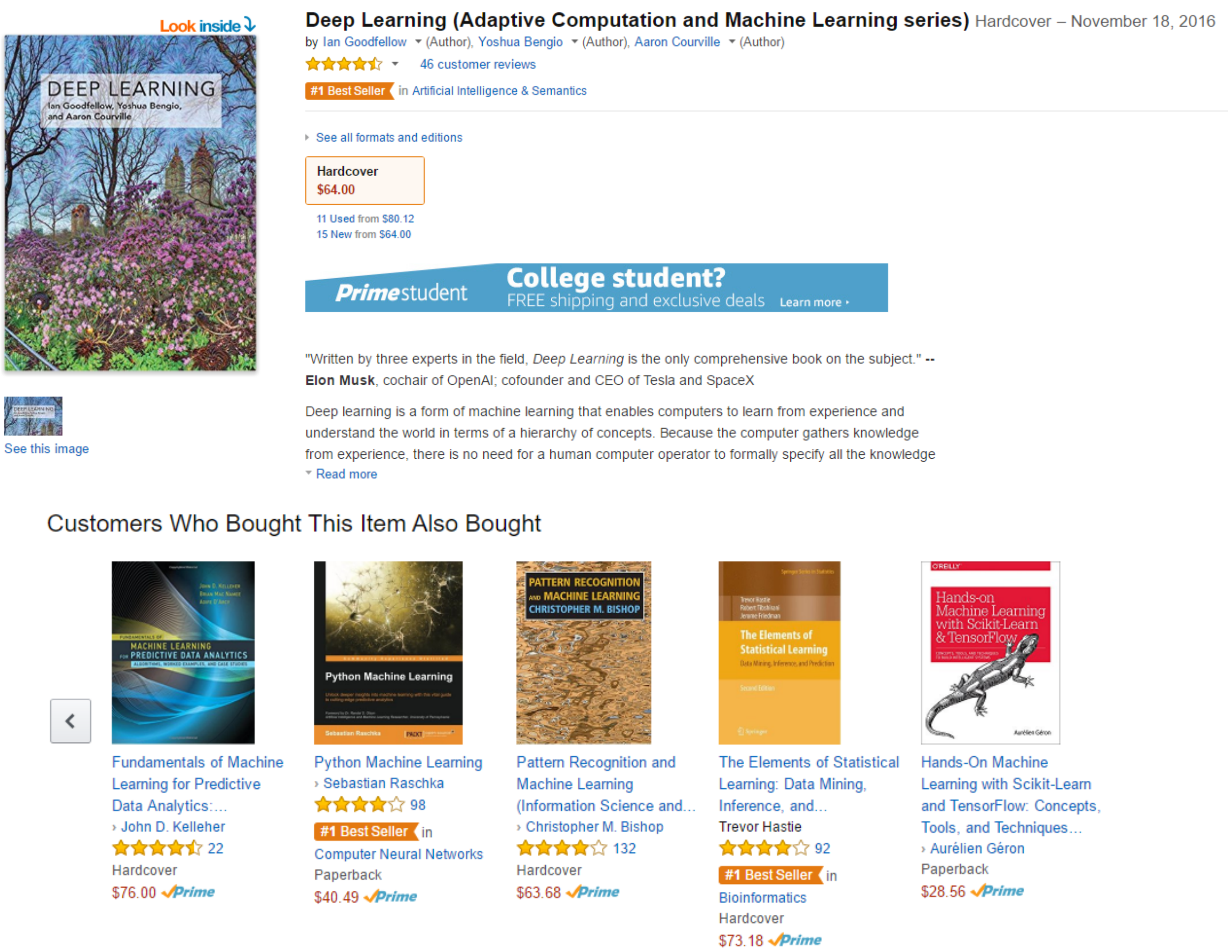
- One-hot encoding
- All features are with latent factors
- A more general regression model

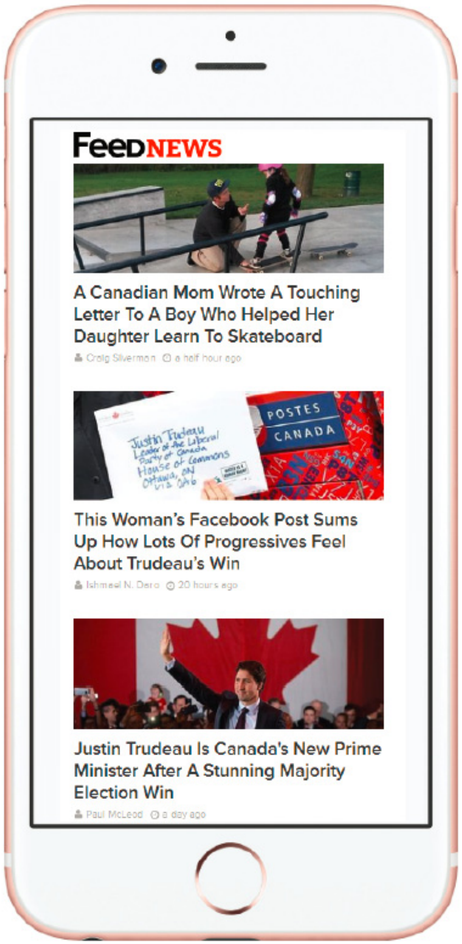
Steffen Rendle. Factorization Machines. ICDM 2010
http://www.ismll.uni-hildesheim.de/pub/pdfs/Rendle2010FM.pdf
Open source: http://www.libfm.org/

Beyond Rating Prediction

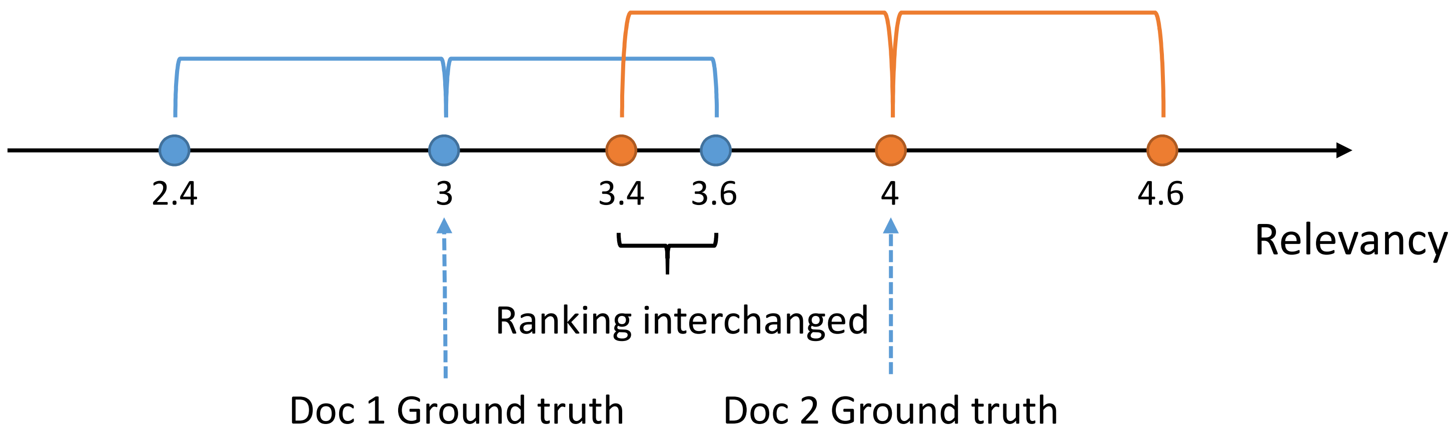
LambdaRank CF

Recommendation is always rendered by ranking





Rating Prediction vs. Ranking



- Same RMSE/MAE might lead to different rankings

Learning to Rank in Collaborative Filtering

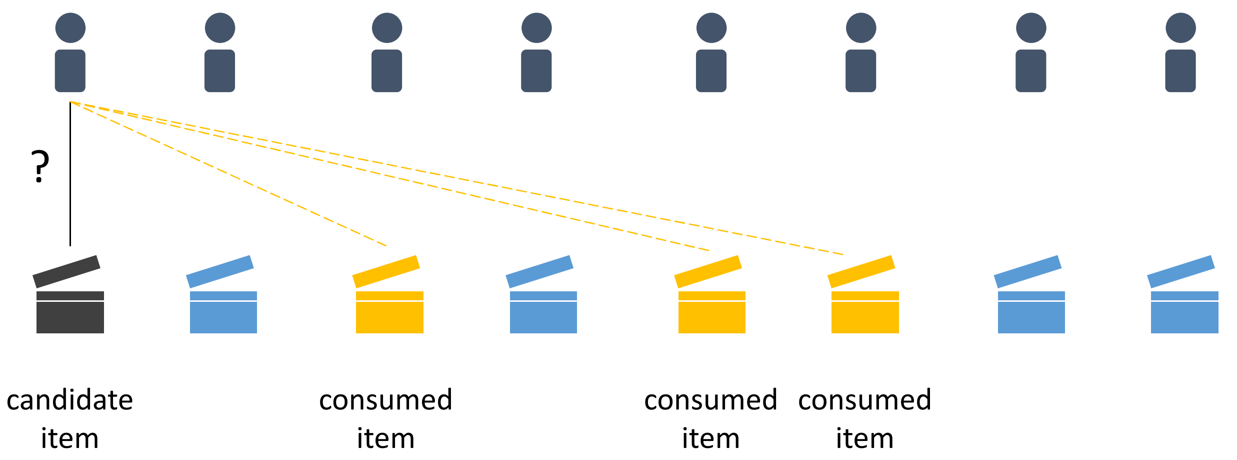
- Previous work on rating prediction can be regarded as pointwise approaches in CF
  - MF, FM, kNN, MF with temporal dynamics and neighborhood information etc.

- Pairwise approaches in CF
  - Bayesian personalized ranking (BPR)

- Listwise approaches in CF
  - LambdaRank CF, LambdaFM

Implicit Feedback Data

- No explicit preference, e.g. rating, shown in the user-item interaction
  - Only clicks, share, comments etc.



Bayesian Personalized Ranking (BPR)

- Basic latent factor model (MF) for scoring

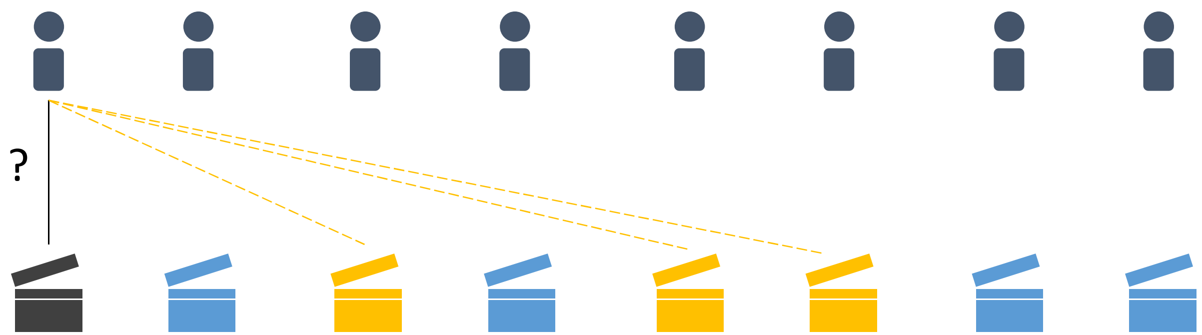
$$\hat{r}_{u,i} = \mu + b_u + b_i + p_u^\top q_i$$

- The (implicit feedback) training data for each user *u*

$$D_u = \{\langle i, j \rangle_u | i \in I_u \wedge j \in I \backslash I_u\}$$

consumed item

unconsumed item



Rendle, Steffen, et al. "BPR: Bayesian personalized ranking from implicit feedback." *UAI*, 2009.

Bayesian Personalized Ranking (BPR)

- Loss function on the ranking prediction of <*i,j*>$_u$

$$\mathcal{L}(\langle i,j\rangle_u) = z_u \cdot \frac{1}{1+\exp(\hat{r}_{u,i}-\hat{r}_{u,j})}$$

Normalizer   Inverse logistic loss

- Gradient

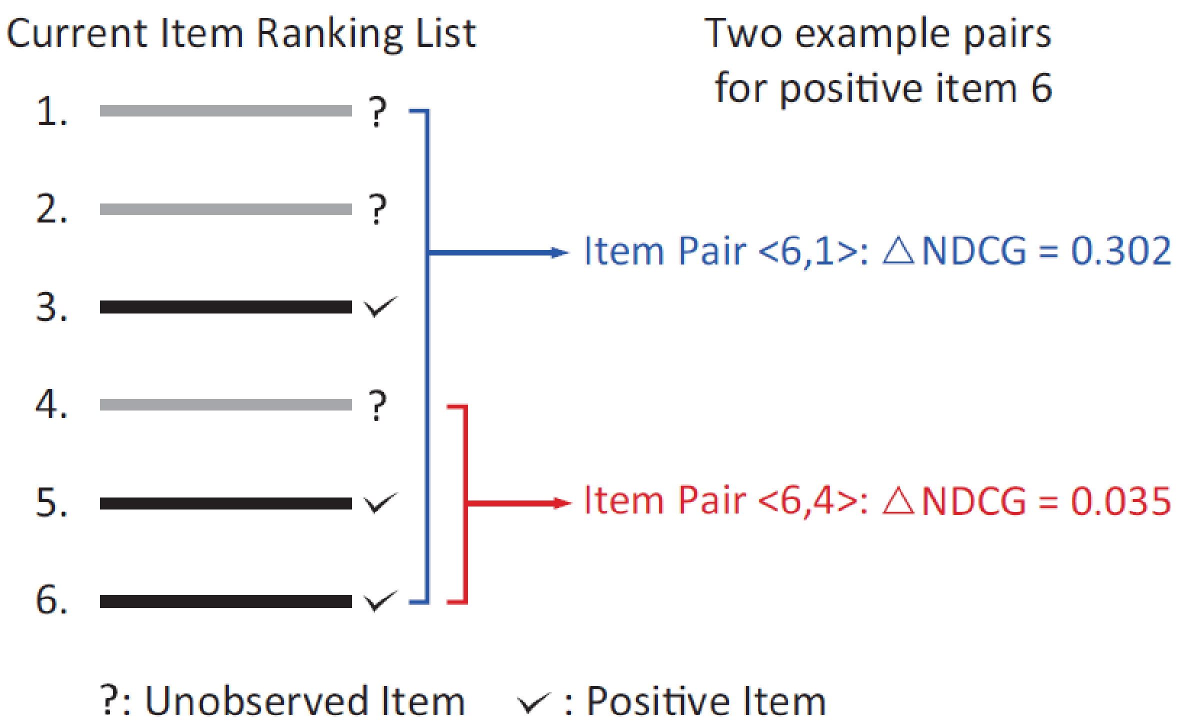
$$\frac{\partial \mathcal{L}(\langle i,j\rangle_u)}{\partial \theta} = \frac{\partial \mathcal{L}(\langle i,j\rangle_u)}{\partial (\hat{r}_{u,i}-\hat{r}_{u,j})} \frac{\partial (\hat{r}_{u,i}-\hat{r}_{u,j})}{\partial \theta}$$

$$\equiv \lambda_{i,j}\left(\frac{\partial \hat{r}_{u,i}}{\partial \theta} - \frac{\partial \hat{r}_{u,j}}{\partial \theta}\right)$$

Rendle, Steffen, et al. "BPR: Bayesian personalized ranking from implicit feedback." *UAI*, 2009.

LambdaRank CF

- Use the idea of LambdaRank to optimise ranking performance in recommendation tasks



Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

Recommendation vs. Web Search

- Difference between them
  - Recommender system should rank all the items
    - Usually more than 10k
  - Search engine only ranks a small subset of retrieved documents
    - Usually fewer than 1k

- For each training iteration, LambdaRank needs the model to rank all the items to get $\Delta NDCG_{i,j}$, super large complexity

Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

LambdaRank CF Solution

- Idea: to generate the item pairs with the probability proportional to their lambda

$$\frac{\partial \mathcal{L}(\langle i,j\rangle_u)}{\partial \theta} = f(\lambda_{i,j},\zeta_u)\Big(\frac{\partial \hat{r}_{u,i}}{\partial \theta} - \frac{\partial \hat{r}_{u,j}}{\partial \theta}\Big)$$

$$f(\lambda_{i,j},\zeta_u) \equiv \lambda_{i,j}\Delta NDCG_{i,j}$$

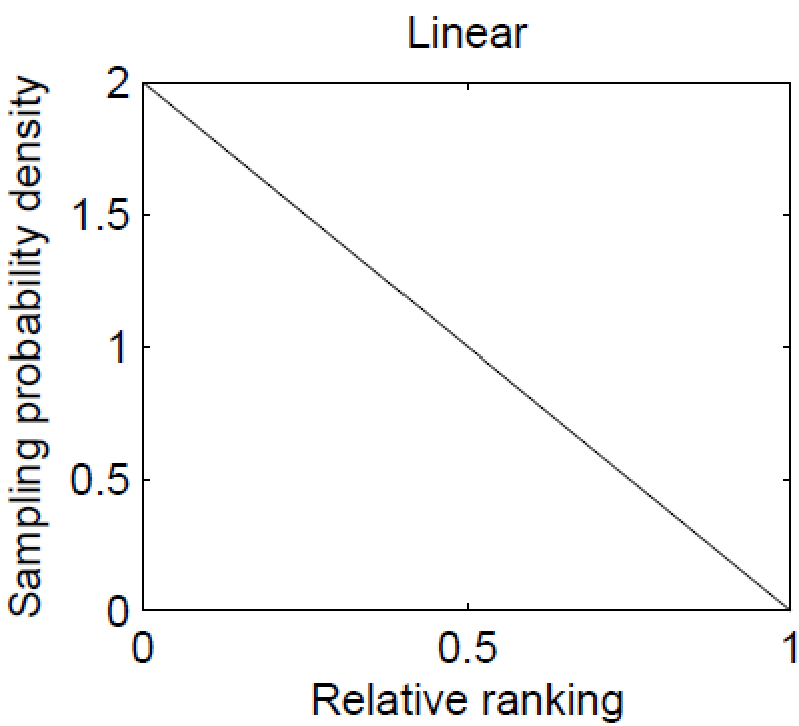
$$p_j \propto f(\lambda_{i,j},\zeta_u)/\lambda_{i,j}$$

- $x_i \in [0,1]$ is the relative ranking position
  - 0 means ranking at top, 1 means ranking at tail

Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

Different Sampling Methods

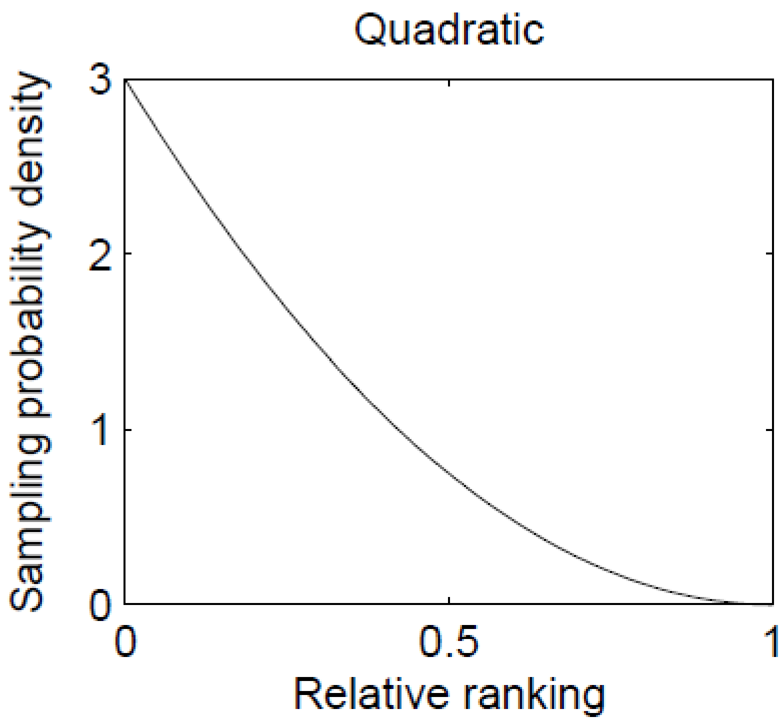
- For each positive item, find 2 candidate items, then choose the one with higher prediction score as the negative item.



Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

Different Sampling Methods

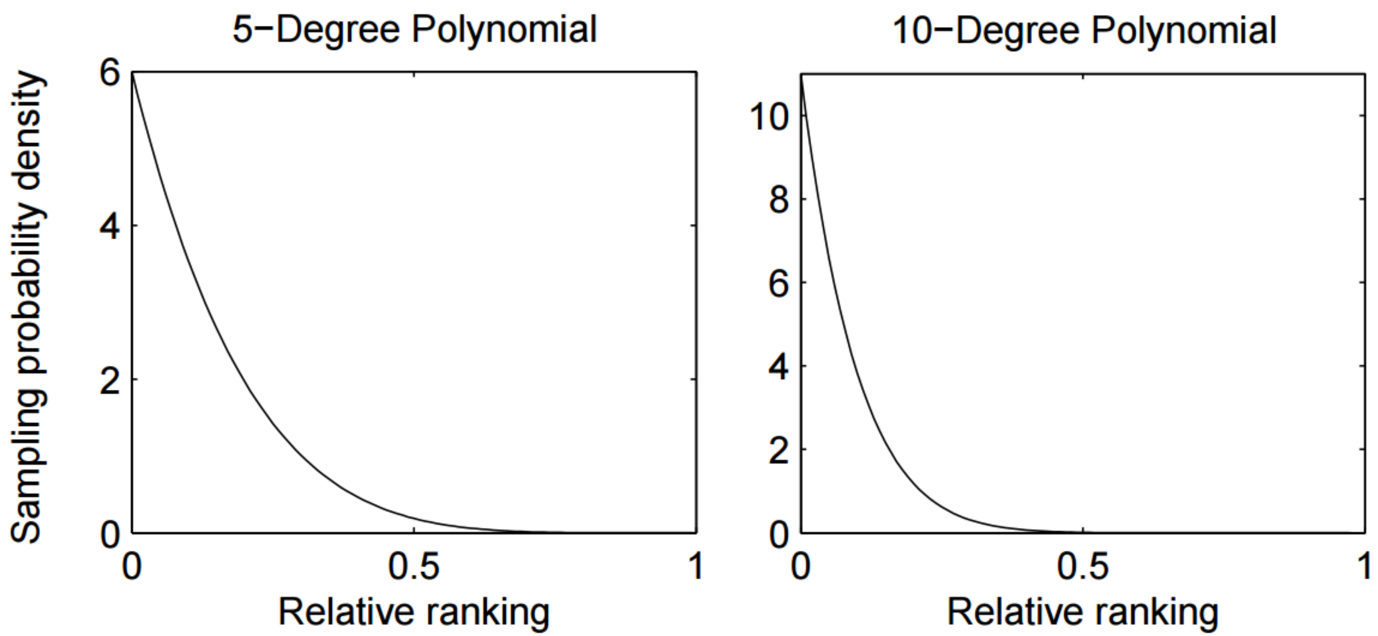
- For each positive item, find **3** candidate items, then choose the one with the **highest** prediction score as the negative item.



Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

Different Sampling Methods

- For each positive item, find ***k*** candidate items, then choose the one with the **highest** prediction score as the negative item.



Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

Experiments on Top-N Recommendation

- Top-N recommendation on 3 datasets

Dataset	Netflix	Yahoo! Music	Last.fm
Users	480,189	1,000,990	992
Items	17,770	624,961	961,417
Ratings	100,480,507	262,810,175	19,150,868

- Performance (DNS is our LambdaCF algorithm)

Netflix

	P@5	P@10	NDCG@5	NDCG@10	MAP
BPR	0.3826	0.3272	0.2052	0.2017	0.1403
DNS	0.4708	0.4012	0.2906	0.2887	0.2036
Impv.	23.1%*	22.6%*	41.6%*	43.1%*	45.1%*

Yahoo! Music

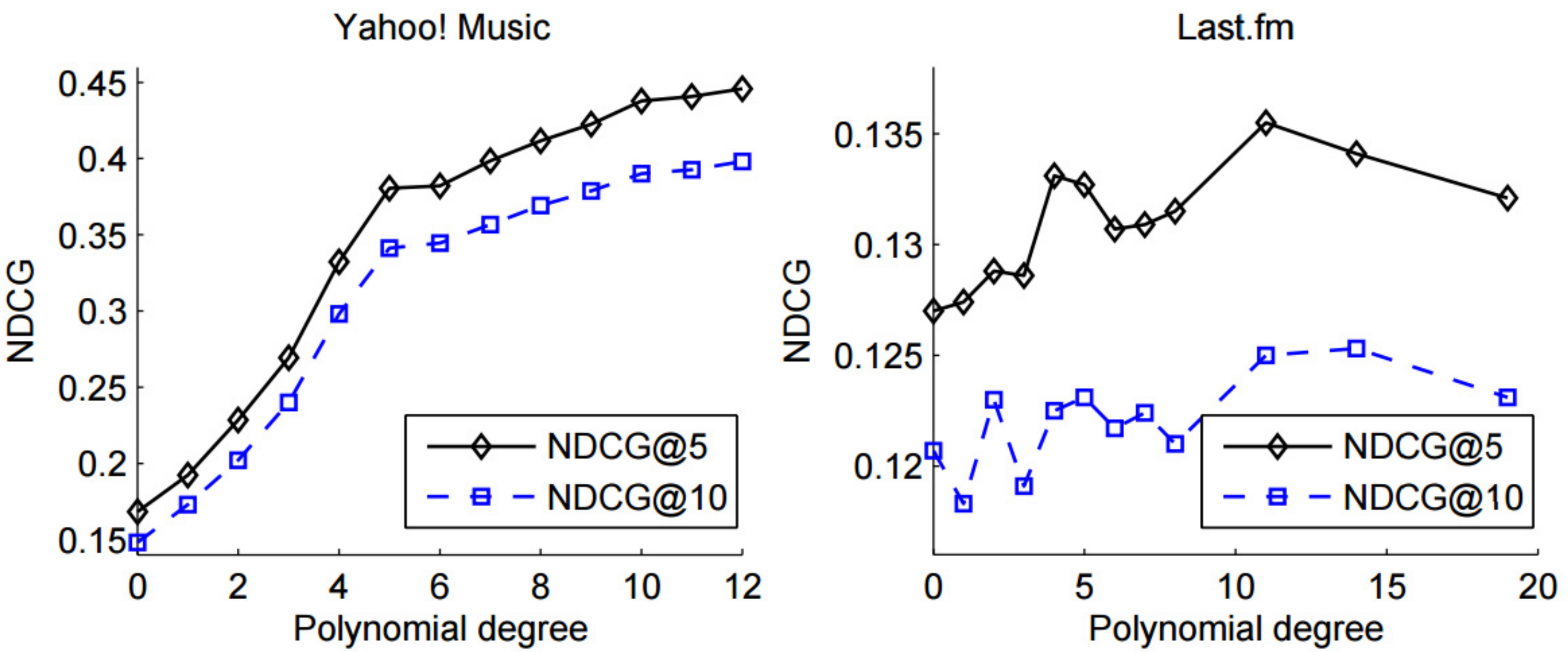
	P@5	P@10	NDCG@5	NDCG@10	MAP
BPR	0.1588	0.1359	0.1683	0.1481	0.0615
DNS	0.4243	0.3671	0.4458	0.3981	0.1644
Impv.	167.2%*	170.1%*	164.9%*	168.8%*	167.3%*

Last.fm

	P@5	P@10	NDCG@5	NDCG@10	MAP
BPR	0.1231	0.1168	0.1270	0.1207	0.0221
DNS	0.1323	0.1202	0.1355	0.1250	0.0223
Impv.	7.5%*	2.9%	6.7%*	3.6%	0.9%

Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

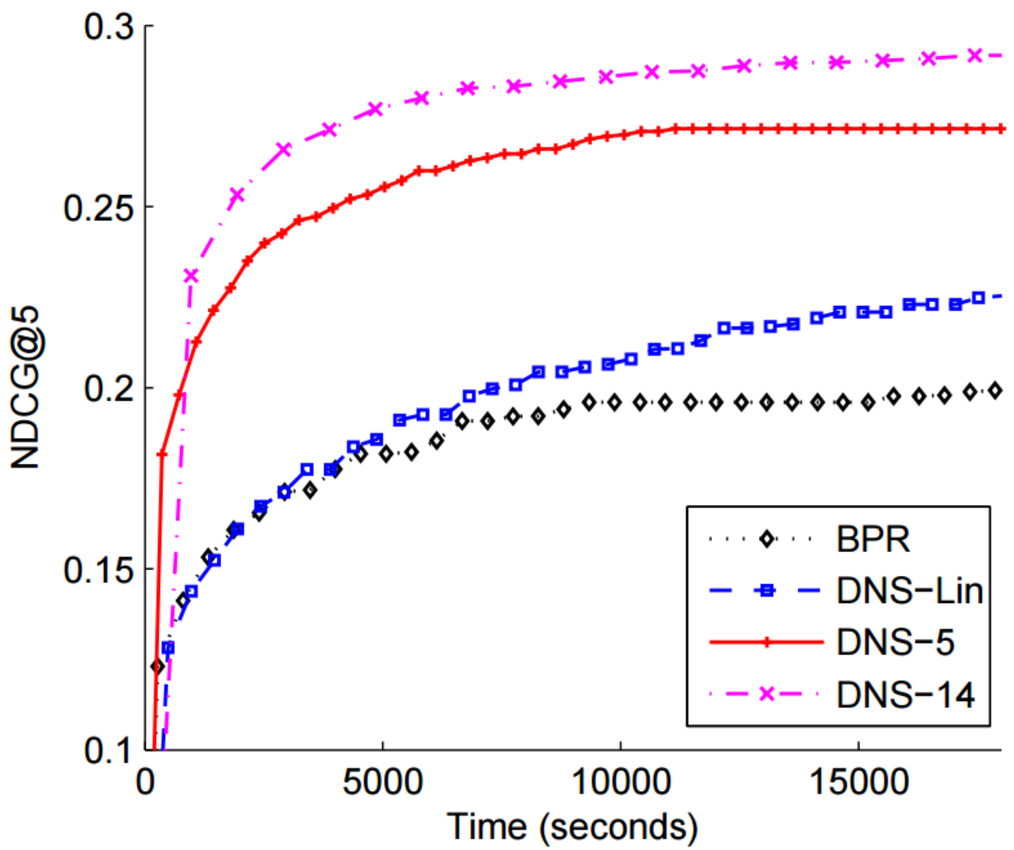
More Empirical Results



- NDCG performance against polynomial degrees on Yahoo! Music and Last.fm datasets

Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

More Empirical Results



Performance convergence against training time on Netflix.

Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.