

CS420 Machine Learning, Lecture 4

Neural Networks

Weinan Zhang

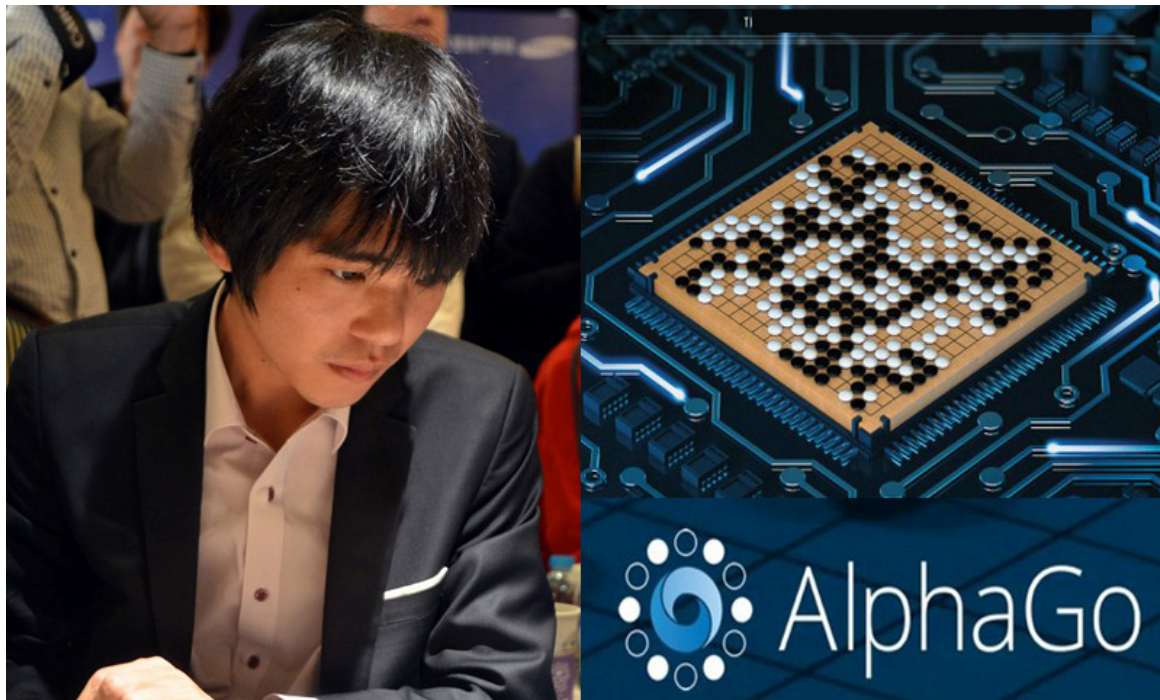
Shanghai Jiao Tong University

<http://wnzhang.net>

<http://wnzhang.net/teaching/cs420/index.html>

Breaking News of AI in 2016

- AlphaGo wins Lee Sedol (4-1)



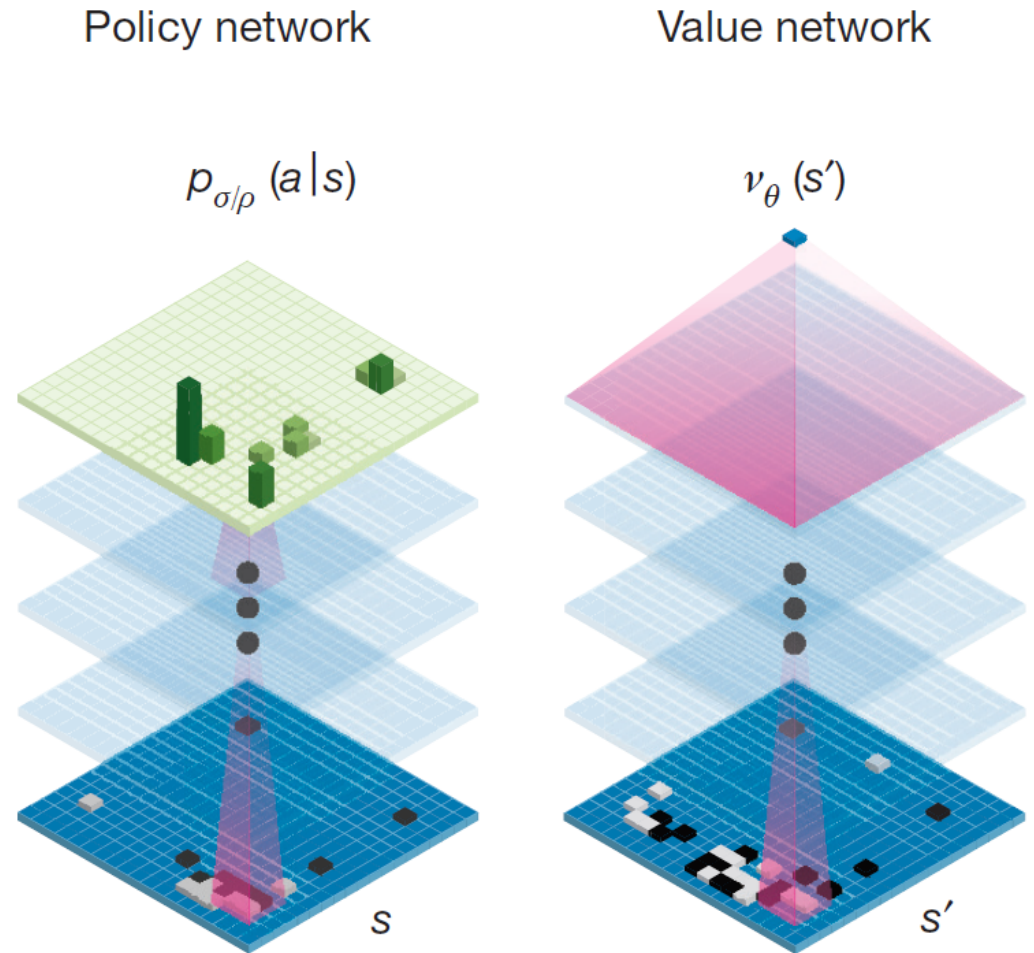
<https://deepmind.com/research/alphago/>

Rank	Name	↑ ↓	Flag	Elo
1	Ke Jie	↑		3628
2	AlphaGo			3598
3	Park Junghwan	↑		3585
4	Tuo Jiaxi	↑		3535
5	Mi Yuting	↑		3534
6	Iyama Yuta	↑		3525
7	Shi Yue	↑		3522
8	Lee Sedol	↑		3521
9	Zhou Ruiyang	↑		3517
10	Shin Jinseok	↑		3503
11	Chen Yaoye	↑		3495
12	Lian Xiao	↑		3493
13	Tan Xiao	↑		3489
14	Kim Iiseok	↑		3489
15	Choi Cheolhan	↑		3482
16	Park Yeonghun	↑		3482
17	Gu Zihao	↑		3468
18	Fan Yunruo	↑		3468
19	Huang Yunsong	↑		3467
20	Li Qincheng	↑		3465
21	Tang Weixing	↑		3461
22	Lee Donghoon	↑		3460
23	Lee Yeongkyu	↑		3459
24	Fan Tingyu	↑		3459
25	Tong Mengcheng	↑		3447
26	Kang Dongyun	↑		3442
27	Wang Xi	↑		3439
28	Weon Seongiin	↑		3439
29	Yang Dingxin	↑		3439
30	Gu Li	↑		3436

<https://www.goratings.org/>

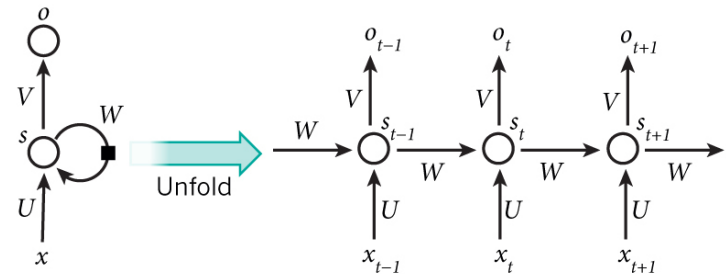
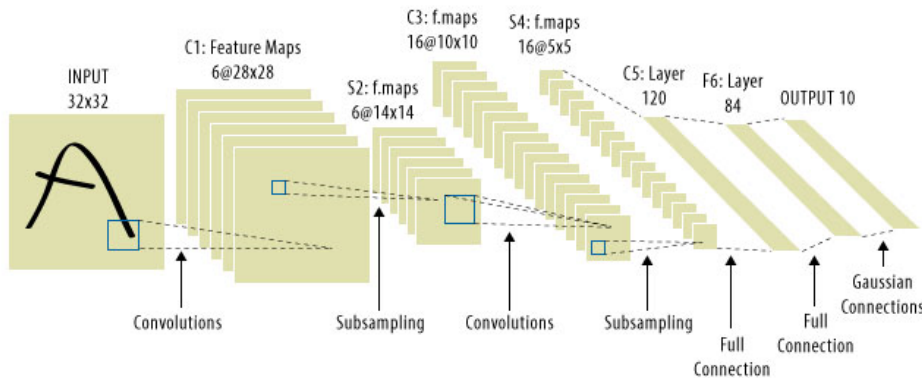
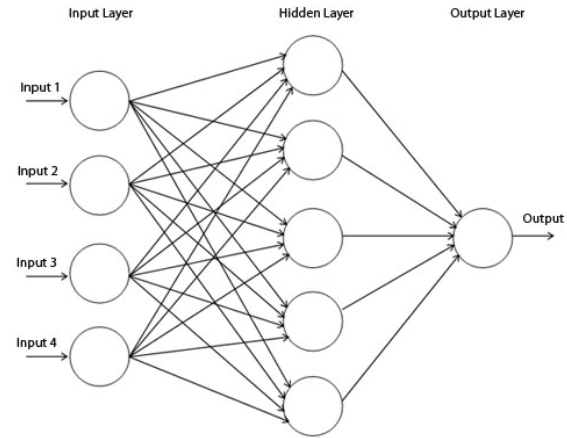
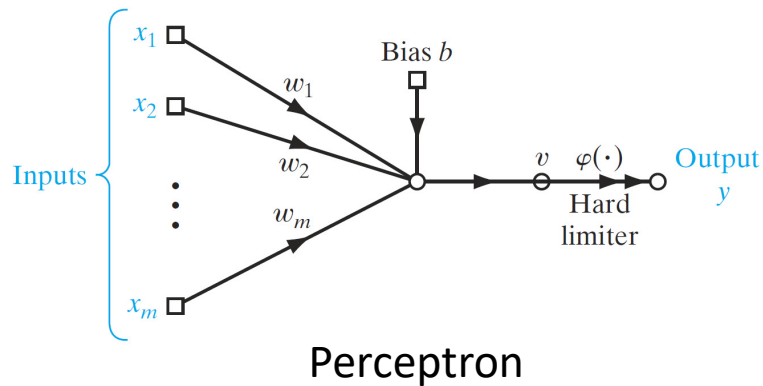
Machine Learning in AlphaGo

- Policy Network
 - Supervised Learning
 - Predict what is the best next human move
 - Reinforcement Learning
 - Learning to select the next move to maximize the winning rate
- Value Network
 - Expectation of winning given the board state
- Implemented by (deep) neural networks



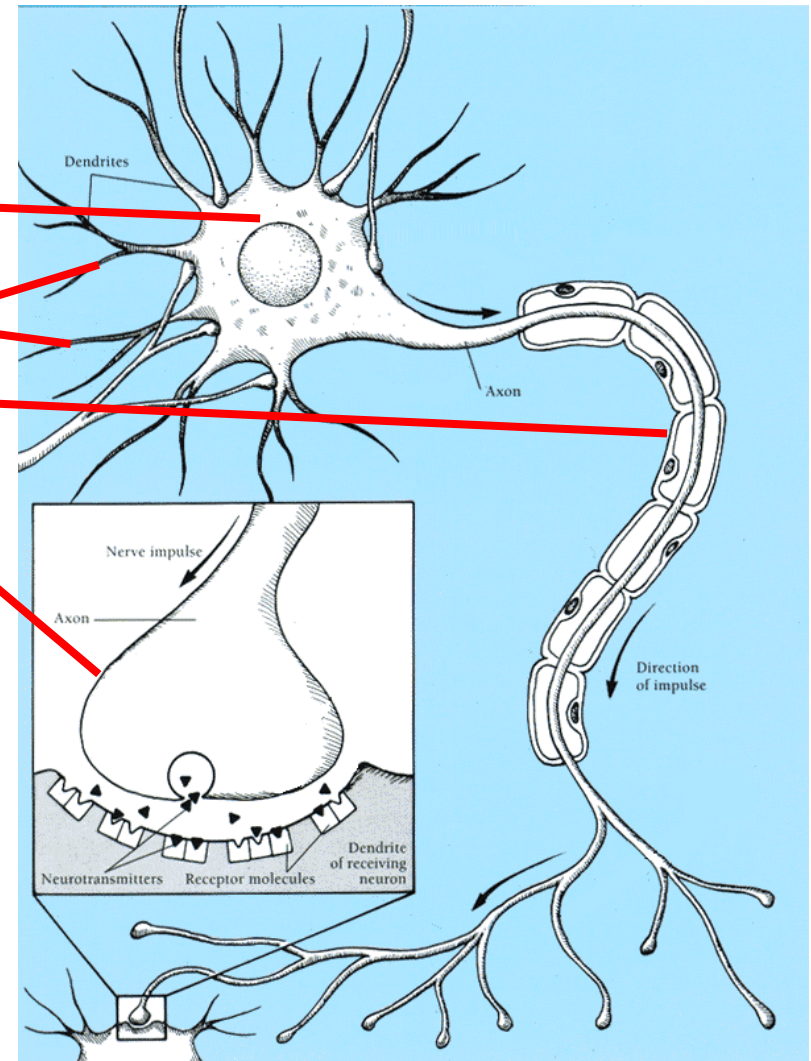
Neural Networks

- Neural networks are the basis of deep learning



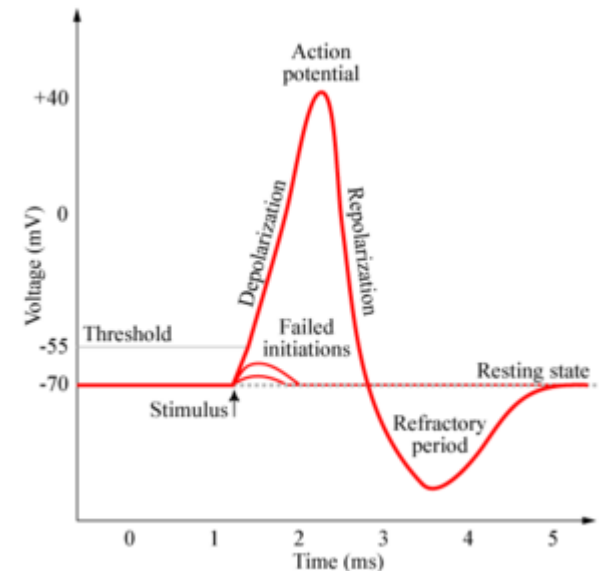
Real Neurons

- Cell structures
 - Cell body
 - Dendrites
 - Axon
 - Synaptic terminals



Neural Communication

- Electrical potential across cell membrane exhibits spikes called action potentials.
- Spike originates in cell body, travels down axon, and causes synaptic terminals to release neurotransmitters.
- Chemical diffuses across synapse to dendrites of other neurons.
- Neurotransmitters can be excitatory or inhibitory.
- If net input of neurotransmitters to a neuron from other neurons is excitatory and exceeds some threshold, it fires an action potential.



Real Neural Learning

- Synapses change size and strength with experience.
- **Hebbian learning**: When two connected neurons are firing at the same time, the strength of the synapse between them increases.
- “Neurons that fire together, wire together.”
- These motivate the research of artificial neural nets

Brief History of Artificial Neural Nets

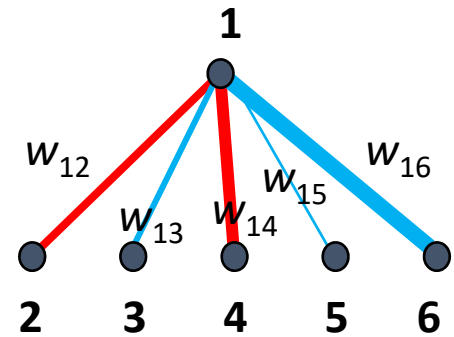
- The First wave
 - 1943 McCulloch and Pitts proposed the [McCulloch-Pitts neuron model](#)
 - 1958 Rosenblatt introduced the simple single layer networks now called [Perceptrons](#).
 - 1969 Minsky and Papert's book Perceptrons demonstrated the limitation of single layer perceptrons, and almost the whole field went into hibernation.
- The Second wave
 - 1986 The [Back-Propagation learning algorithm](#) for Multi-Layer Perceptrons was rediscovered and the whole field took off again.
- The Third wave
 - 2006 [Deep \(neural networks\) Learning](#) gains popularity and
 - 2012 made significant break-through in many applications.

Artificial Neuron Model

- Model network as a graph with cells as nodes and synaptic connections as weighted edges from node i to node j , w_{ji}

- Model net input to cell as

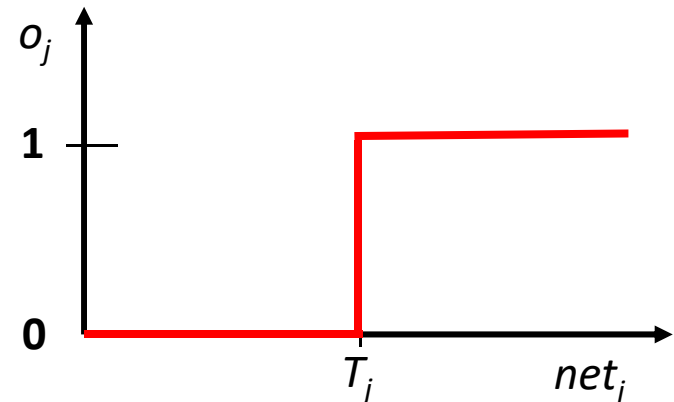
$$\text{net}_j = \sum_i w_{ji} o_i$$



- Cell output is

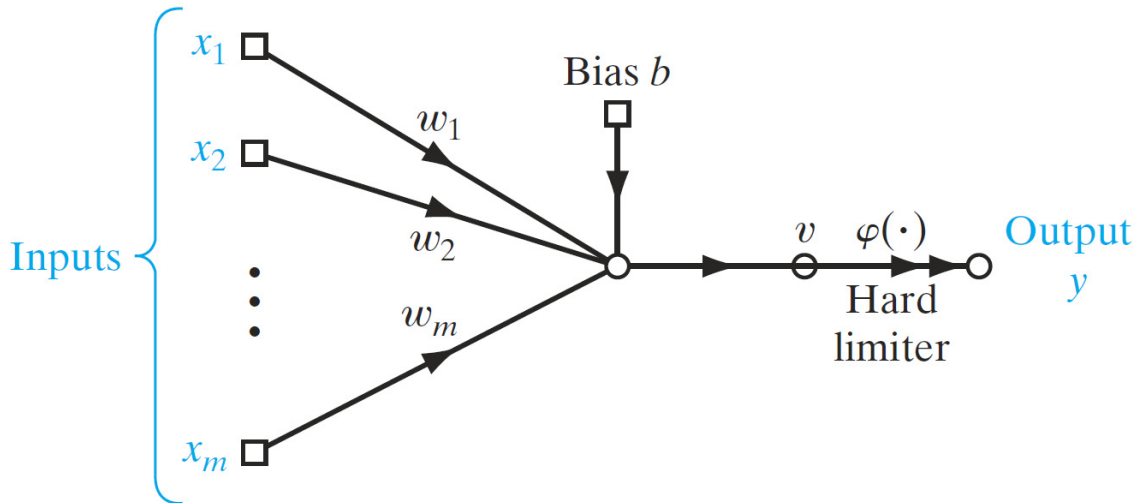
$$o_j = \begin{cases} 0 & \text{if } \text{net}_j < T_j \\ 1 & \text{if } \text{net}_j \geq T_j \end{cases}$$

(T_j is threshold for unit j)



Perceptron Model

- Rosenblatt's single layer perceptron [1958]



- Rosenblatt [1958] further proposed the *perceptron* as the first model for learning with a teacher (i.e., supervised learning)
- Focused on how to find appropriate weights w_m for two-class classification task

- Prediction

$$\hat{y} = \varphi\left(\sum_{i=1}^m w_i x_i + b\right)$$

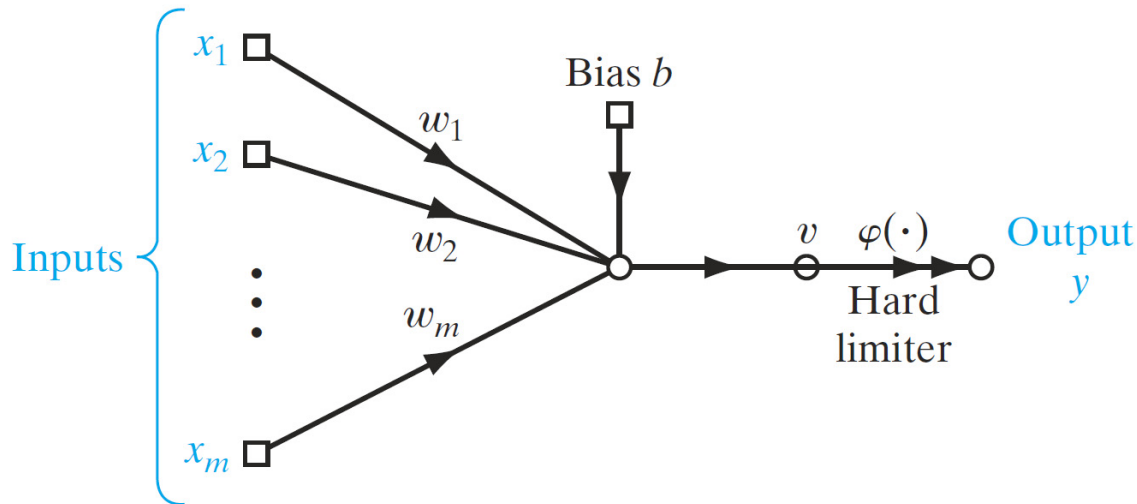
- Activation function

$$\varphi(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

- $y = 1$: class one
- $y = -1$: class two

Training Perceptron

- Rosenblatt's single layer perceptron [1958]



- Training

$$w_i = w_i + \eta(y - \hat{y})x_i$$

$$b = b + \eta(y - \hat{y})$$

- Equivalent to rules:

- If output is correct, do nothing
- If output is high, lower weights on positive inputs
- If output is low, increase weights on active inputs

- Prediction

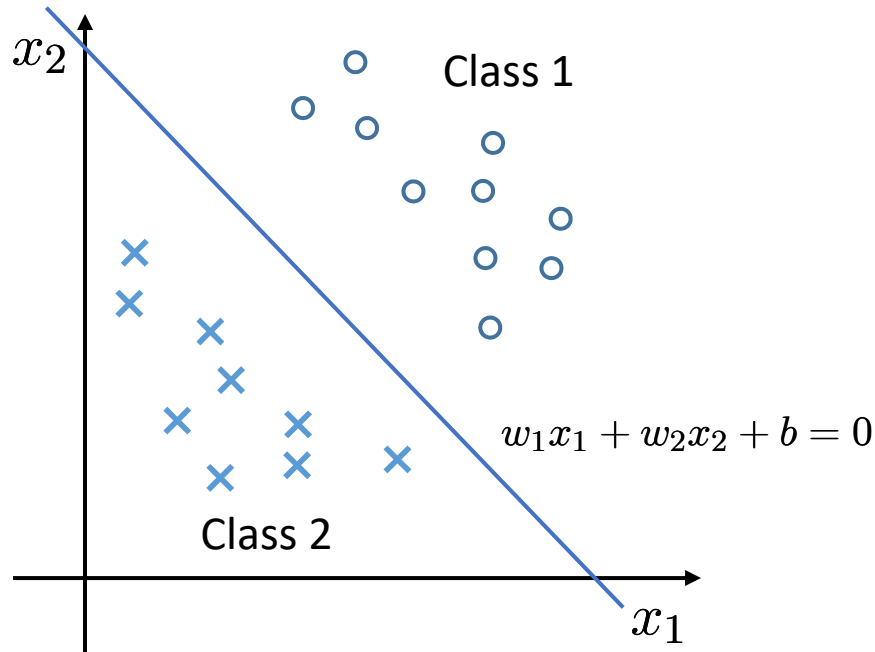
$$\hat{y} = \varphi\left(\sum_{i=1}^m w_i x_i + b\right)$$

- Activation function

$$\varphi(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

Properties of Perceptron

- Rosenblatt's single layer perceptron [1958]

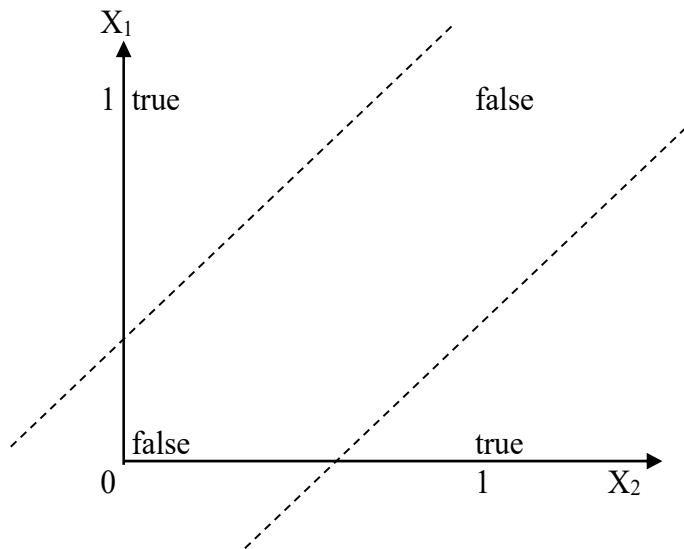


- Rosenblatt Proved the convergence of a learning algorithm if two classes said to be linearly separable (i.e., patterns that lie on opposite sides of a hyperplane)
- Many people hoped that such a machine could be the basis for artificial intelligence

Properties of Perceptron

- The XOR problem

Input x		Output y
X_1	X_2	$X_1 \text{ XOR } X_2$
0	0	0
0	1	1
1	0	1
1	1	0

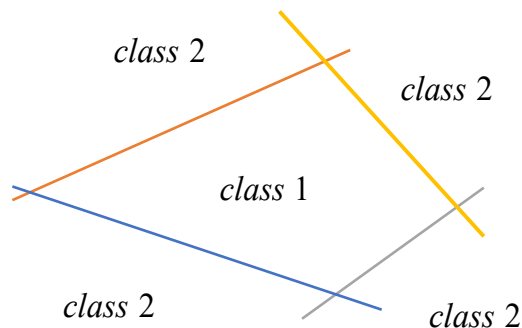
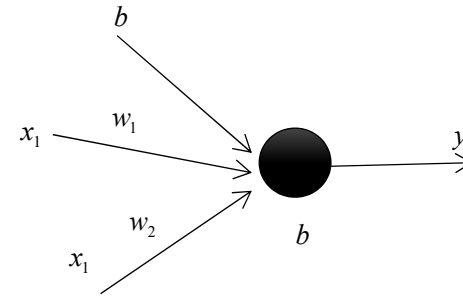
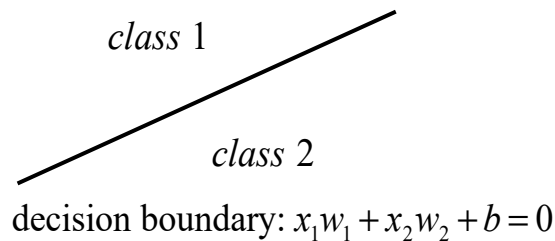


XOR is non linearly separable: These two classes (true and false) cannot be separated using a line.

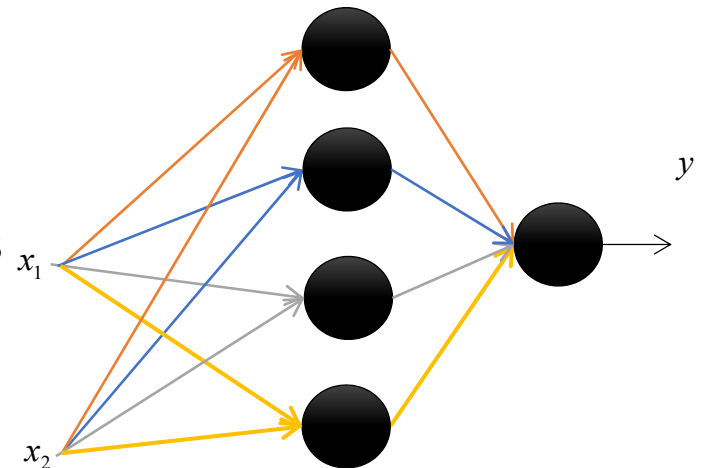
- However, Minsky and Papert [1969] showed that some rather elementary computations, such as *XOR* problem, could not be done by Rosenblatt's one-layer perceptron
- However Rosenblatt believed the limitations could be overcome if more layers of units to be added, but no learning algorithm known to obtain the weights yet
- Due to the lack of learning algorithms people left the neural network paradigm for almost 20 years

Hidden Layers and Backpropagation (1986~)

- Adding hidden layer(s) (internal presentation) allows to learn a mapping that is not constrained by **linearly separable**



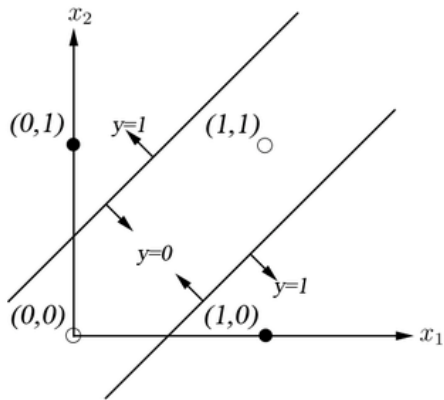
Each hidden node realizes one of the lines bounding the convex region



Hidden Layers and Backpropagation (1986~)

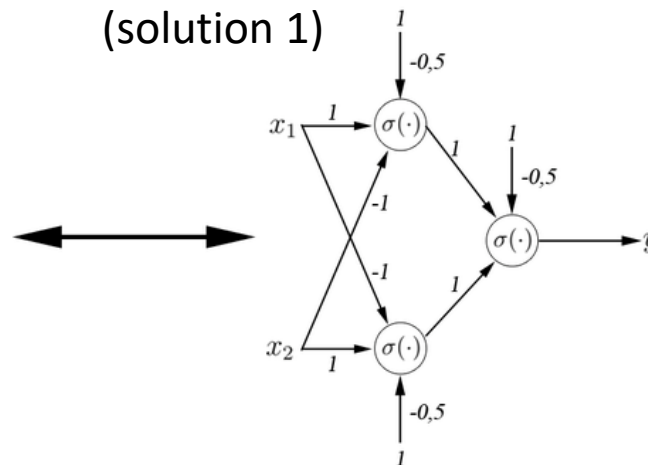
- But the solution is quite often not unique

Input x		Output y
X_1	X_2	$X_1 \text{ XOR } X_2$
0	0	0
0	1	1
1	0	1
1	1	0



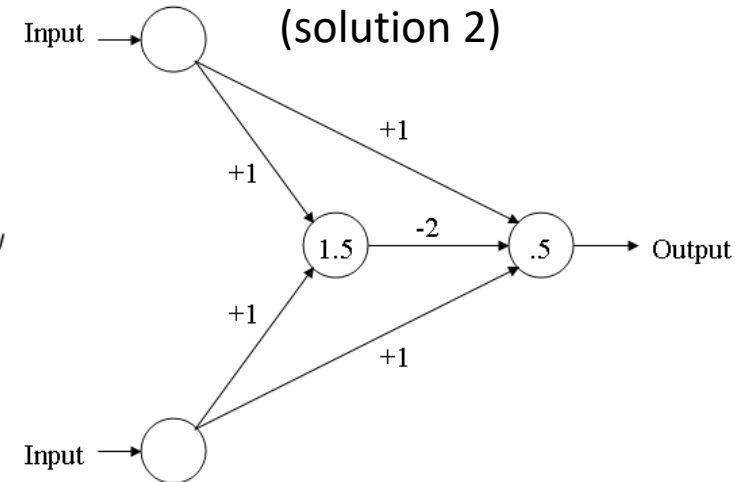
Two lines are necessary to divide the sample space accordingly

(solution 1)



Sign activation function

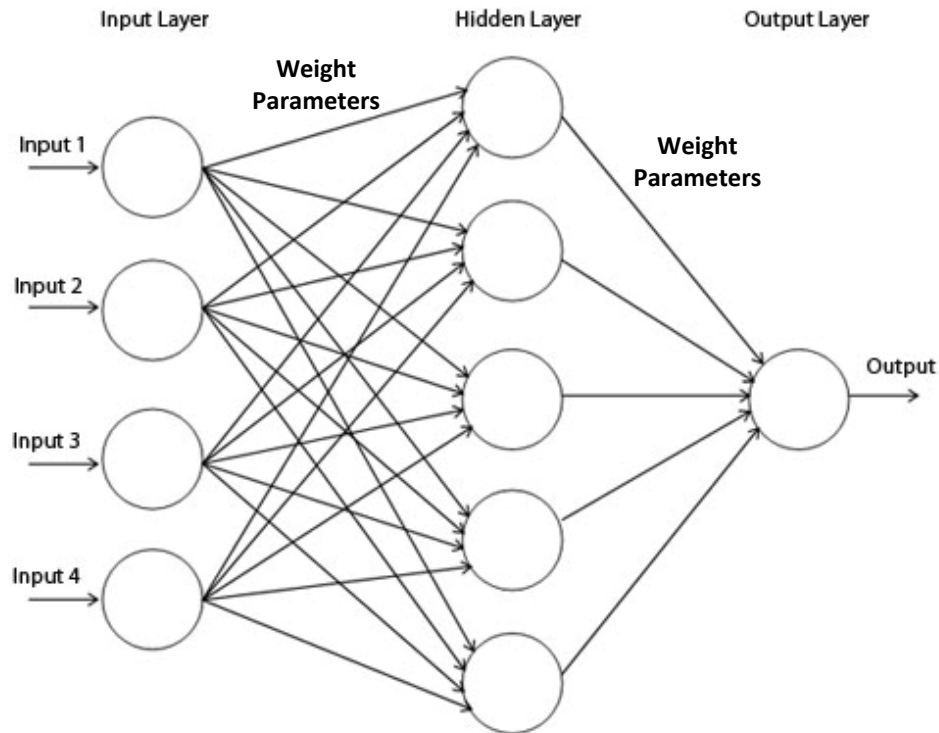
(solution 2)



The number in the circle is a threshold

Hidden Layers and Backpropagation (1986~)

- **Feedforward**: messages move forward from the input nodes, through the hidden nodes (if any), and to the output nodes. There are no cycles or loops in the network

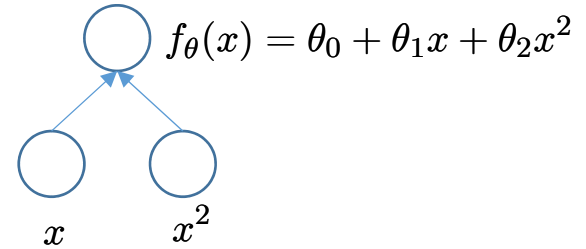


Two-layer feedforward neural network

Single / Multiple Layers of Calculation

- Single layer function

$$f_{\theta}(x) = \sigma(\theta_0 + \theta_1 x + \theta_2 x^2)$$

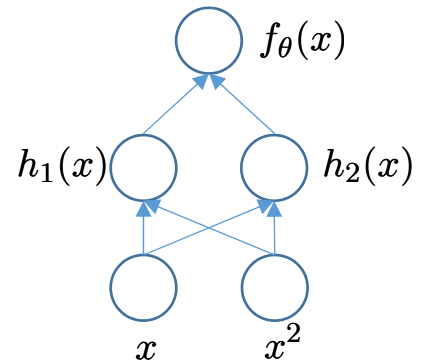


- Multiple layer function

$$h_1(x) = \tanh(\theta_0 + \theta_1 x + \theta_2 x^2)$$

$$h_2(x) = \tanh(\theta_3 + \theta_4 x + \theta_5 x^2)$$

$$f_{\theta}(x) = f_{\theta}(h_1(x), h_2(x)) = \sigma(\theta_6 + \theta_7 h_1 + \theta_8 h_2)$$



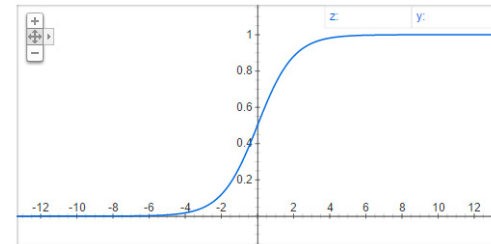
- With non-linear activation function

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad \tanh(x) = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

Non-linear Activation Functions

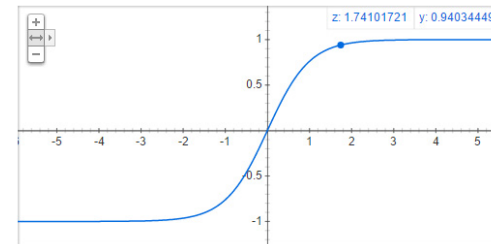
- Sigmoid

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



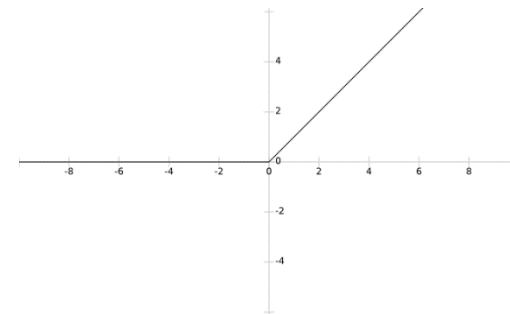
- Tanh

$$\tanh(z) = \frac{1 - e^{-2z}}{1 + e^{-2z}}$$



- Rectified Linear Unit (ReLU)

$$\text{ReLU}(z) = \max(0, z)$$

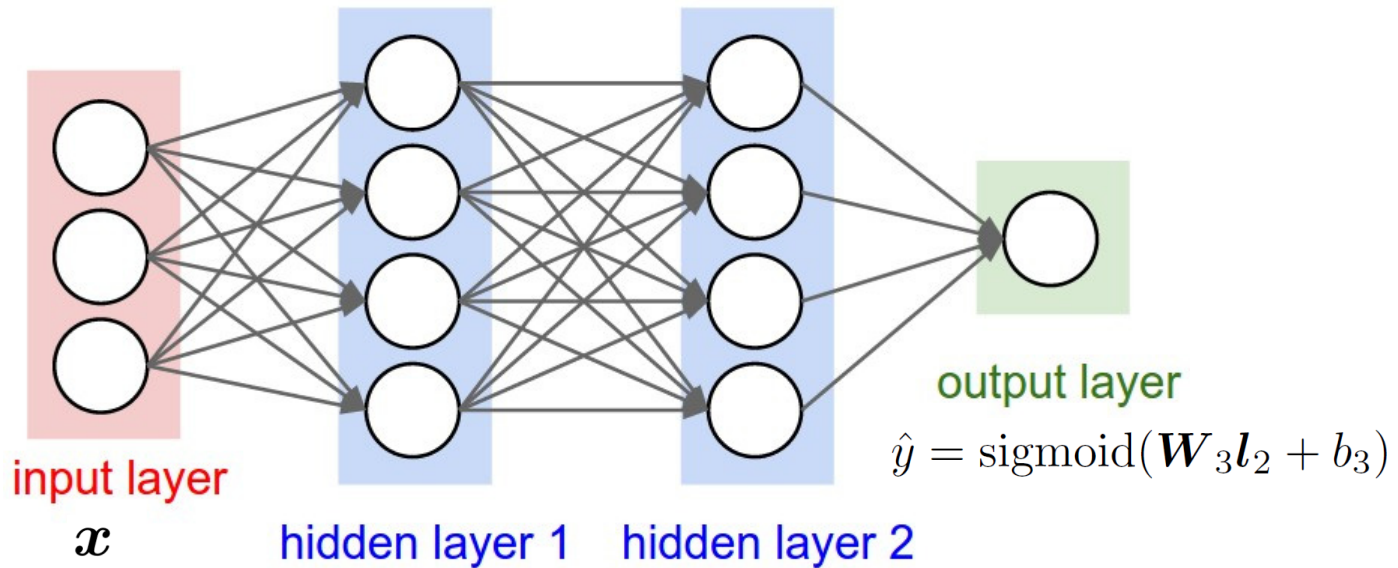


Universal Approximation Theorem

- A feed-forward network with a single hidden layer containing a finite number of neurons (i.e., a multilayer perceptron), can approximate continuous functions
 - on compact subsets of \mathbb{R}^n
 - under mild assumptions on the activation function.
 - Such as Sigmoid, Tanh and ReLU

Universal Approximation

- Multi-layer perceptron approximate any continuous functions on compact subset of \mathbb{R}^n

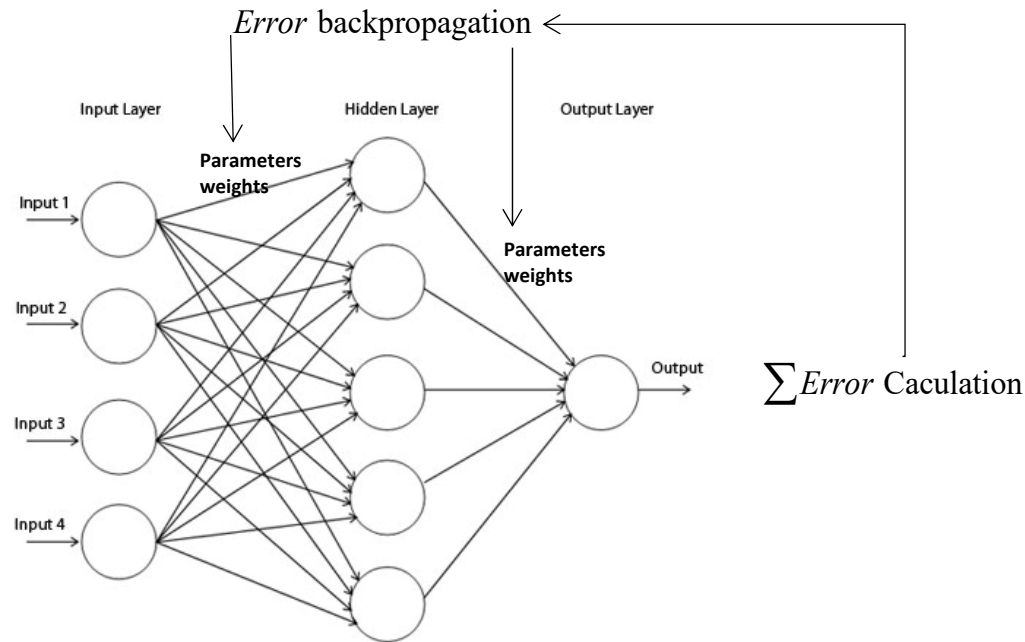


$$l_1 = \tanh(\mathbf{W}_1 x + b_1) \quad l_2 = \tanh(\mathbf{W}_2 l_1 + b_2)$$

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad \tanh(x) = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

Hidden Layers and Backpropagation (1986~)

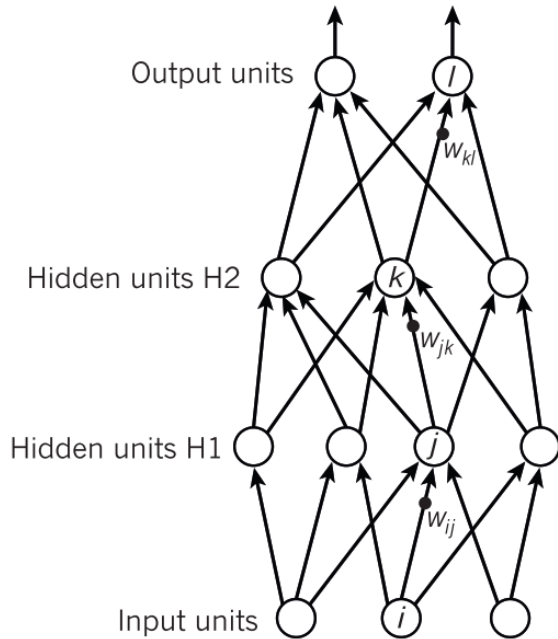
- One of the efficient algorithms for multi-layer neural networks is *the Backpropagation algorithm*
- It was re-introduced in 1986 and Neural Networks regained the popularity



Note: *backpropagation* appears to be found by Werbos [1974]; and then independently rediscovered around 1985 by Rumelhart, Hinton, and Williams [1986] and by Parker [1985]

Learning NN by Back-Propagation

Compare outputs with correct answer to get error



$$y_l = f(z_l)$$

$$z_l = \sum_{k \in H2} w_{kl} y_k$$

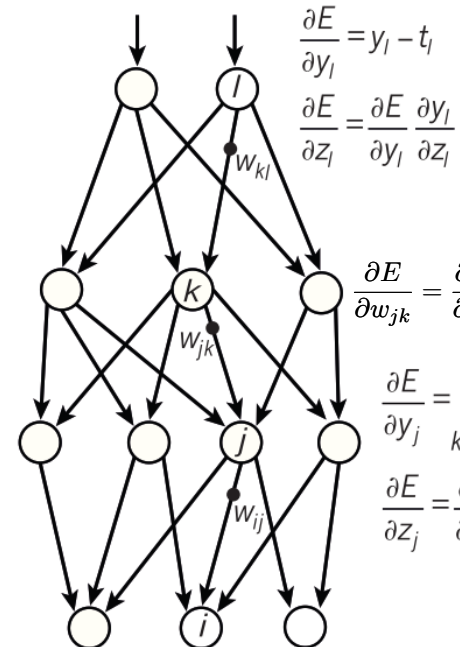
$$y_k = f(z_k)$$

$$z_k = \sum_{j \in H1} w_{jk} y_j$$

$$y_j = f(z_j)$$

$$z_j = \sum_{i \in \text{Input}} w_{ij} x_i$$

Compare outputs with correct answer to get error derivatives



$$\frac{\partial E}{\partial y_l} = y_l - t_l$$

$$\frac{\partial E}{\partial z_l} = \frac{\partial E}{\partial y_l} \frac{\partial y_l}{\partial z_l}$$

$$\frac{\partial E}{\partial y_k} = \sum_{l \in \text{out}} w_{kl} \frac{\partial E}{\partial z_l}$$

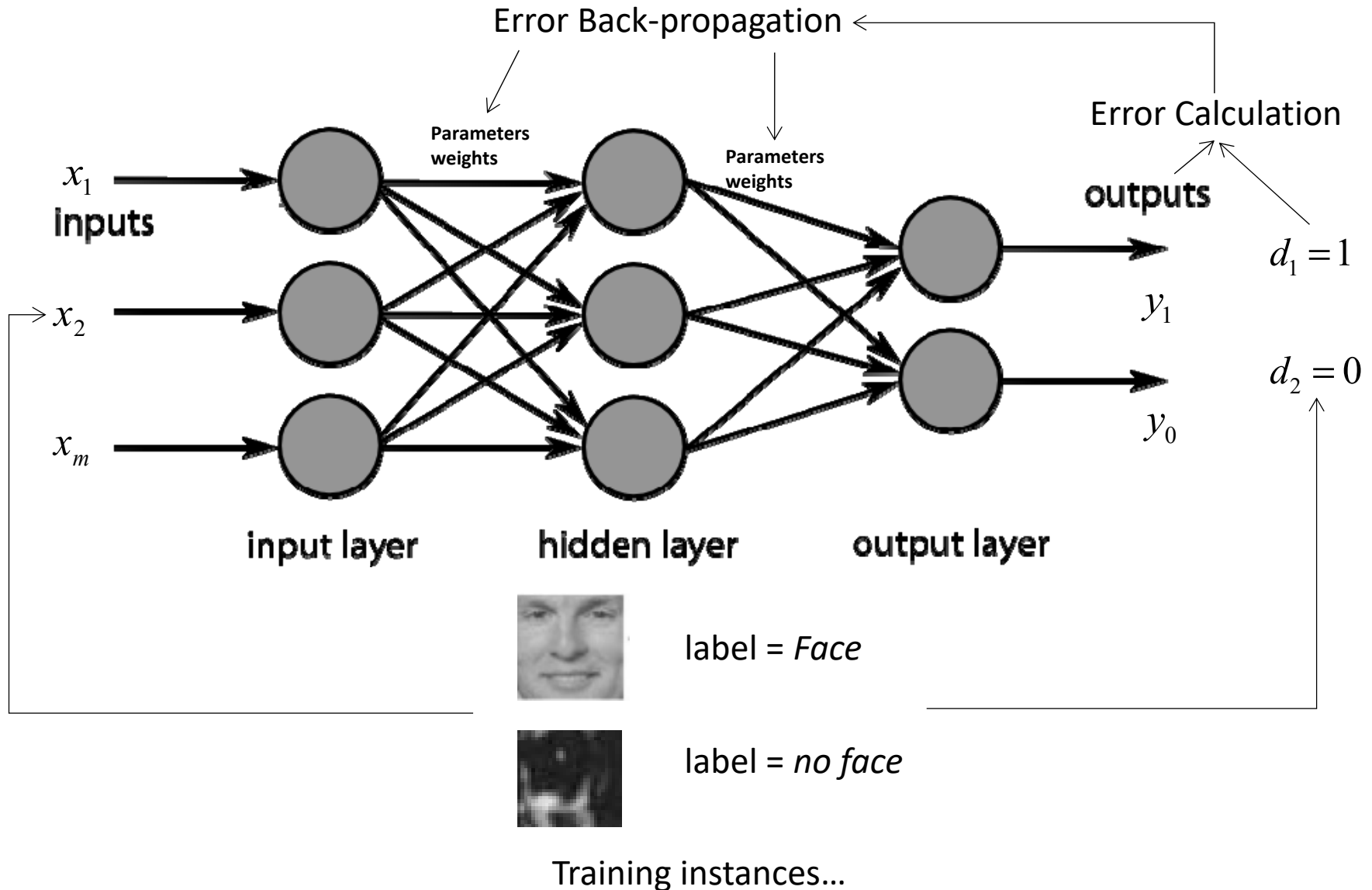
$$\frac{\partial E}{\partial z_k} = \frac{\partial E}{\partial y_k} \frac{\partial y_k}{\partial z_k}$$

$$\frac{\partial E}{\partial w_{jk}} = \frac{\partial E}{\partial z_k} \frac{\partial z_k}{\partial w_{jk}} = \frac{\partial E}{\partial z_k} y_j$$

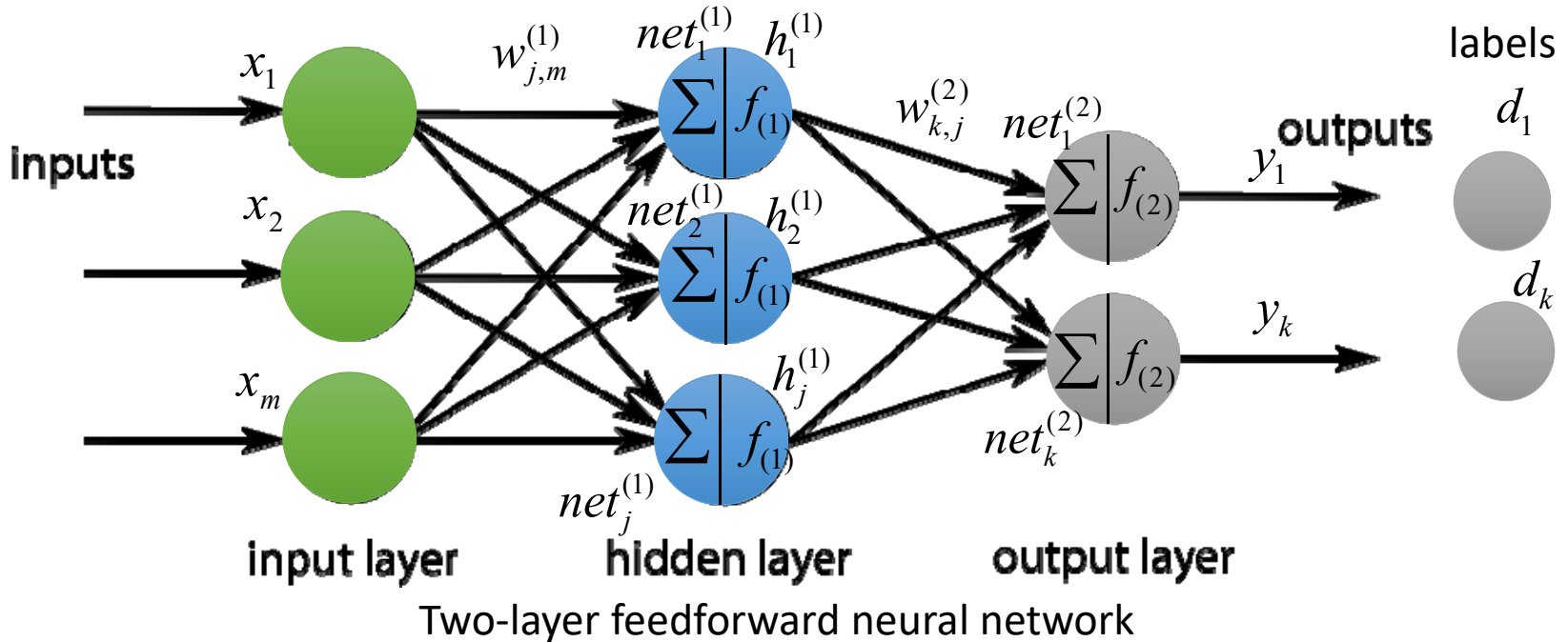
$$\frac{\partial E}{\partial y_j} = \sum_{k \in H2} w_{jk} \frac{\partial E}{\partial z_k}$$

$$\frac{\partial E}{\partial z_j} = \frac{\partial E}{\partial y_j} \frac{\partial y_j}{\partial z_j}$$

Learning NN by Back-Propagation



Make a Prediction



Feed-forward / prediction:

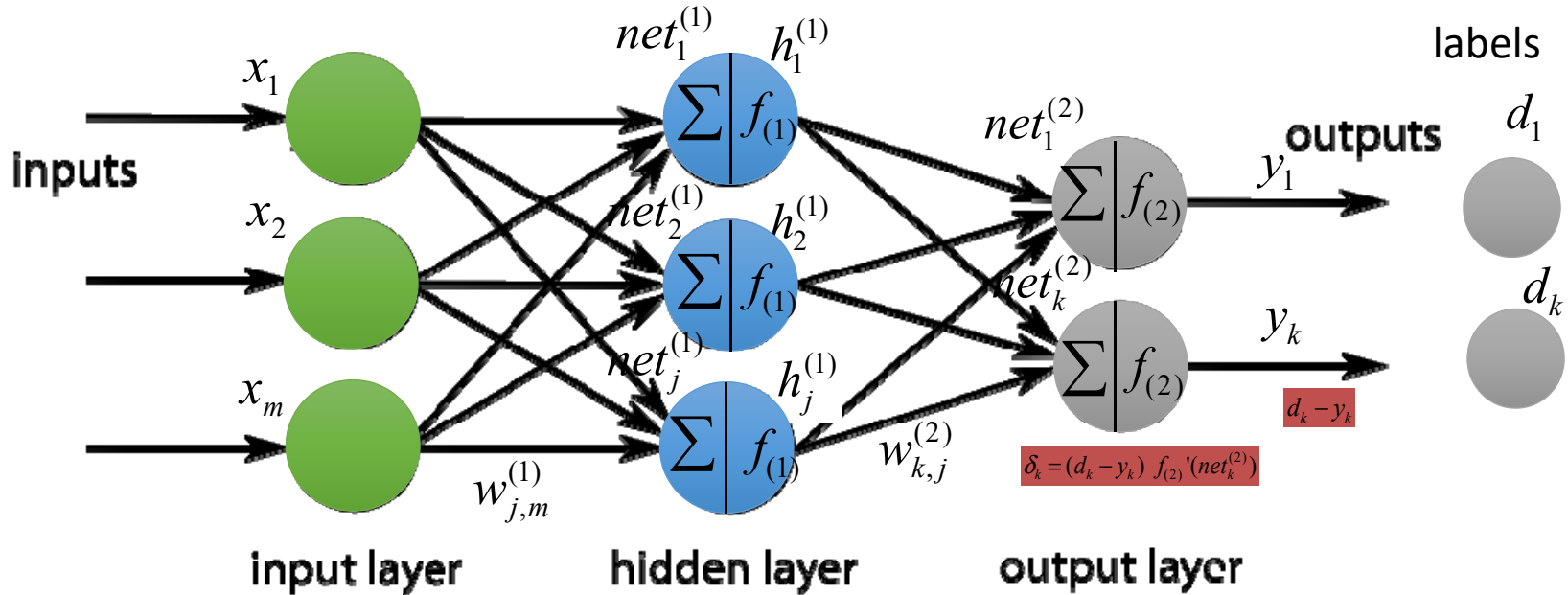
$$x = (x_1, \dots, x_m) \xrightarrow{m} h_j^1 \xrightarrow{j} y_k$$

$$h_j^1 = f_1(\text{net}_j^{(1)}) = f_1\left(\sum w_{j,m}^{(1)} x_m\right) \quad y_k = f_2(\text{net}_k^{(2)}) = f_2\left(\sum w_{k,j}^{(2)} h_j^{(1)}\right)$$

where

$$\text{net}_j^{(1)} \equiv \sum_m w_{j,m}^{(1)} x_m \quad \text{net}_k^{(2)} \equiv \sum_j w_{k,j}^{(2)} h_j^{(1)}$$

When backprop/learn the parameters



Two layer feedforward neural network

Notation:

$$net_j^{(1)} \equiv \sum_m w_{j,m}^{(1)} x_m$$

$$net_k^{(2)} \equiv \sum_j w_{k,j}^{(2)} h_j^{(1)}$$

η : the given learning rate

Backprop to learn the parameters:

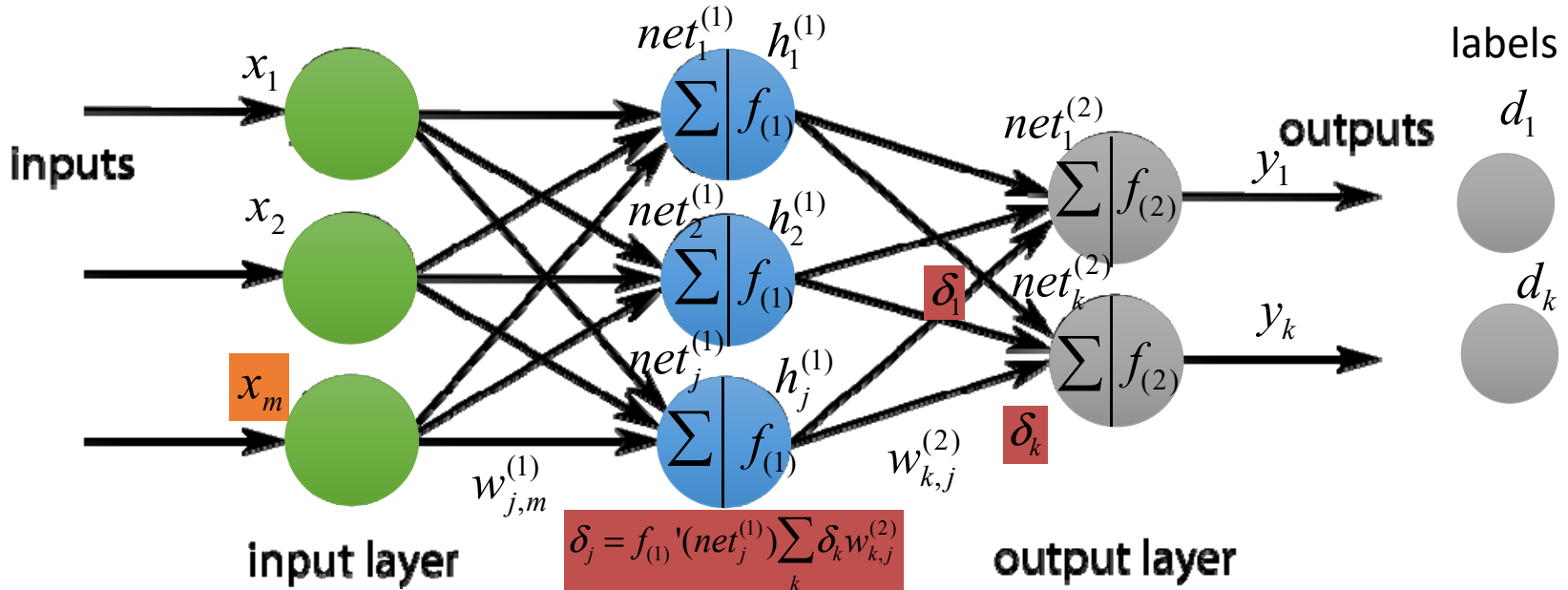
output neuron:

$$w_{k,j}^{(2)} = w_{k,j}^{(2)} + \Delta w_{k,j}^{(2)} \leftarrow \frac{\Delta w_{k,j}^{(2)} = \eta \text{Error}_k \text{Output}_j = \eta \delta_k h_j^{(1)}}{E(W) = \frac{1}{2} \sum_k (y_k - d_k)^2}$$

$$\Delta w_{k,j}^{(2)} \equiv -\eta \frac{\partial E(W)}{\partial w_{k,j}^{(2)}} = -\eta (y_k - d_k) \frac{\partial y_k}{\partial net_k^{(2)}} \frac{\partial net_k^{(2)}}{\partial w_{k,j}^{(2)}} = \eta (d_k - y_k) f_{(2)}'(net_k^{(2)}) h_j^{(1)} = \eta \delta_k h_j^{(1)}$$

$$\text{Error} : \delta_k \equiv (d_k - y_k) f_{(2)}'(net_k^{(2)})$$

Multi-layer feedforward neural network



Two layer feedforward neural network

Notation:

$$net_j^{(1)} \equiv \sum_m w_{j,m}^{(1)} x_m$$

$$net_k^{(2)} \equiv \sum_j w_{k,j}^{(2)} h_j^{(1)}$$

η : the given learning rate

Backprop to learn the parameters:

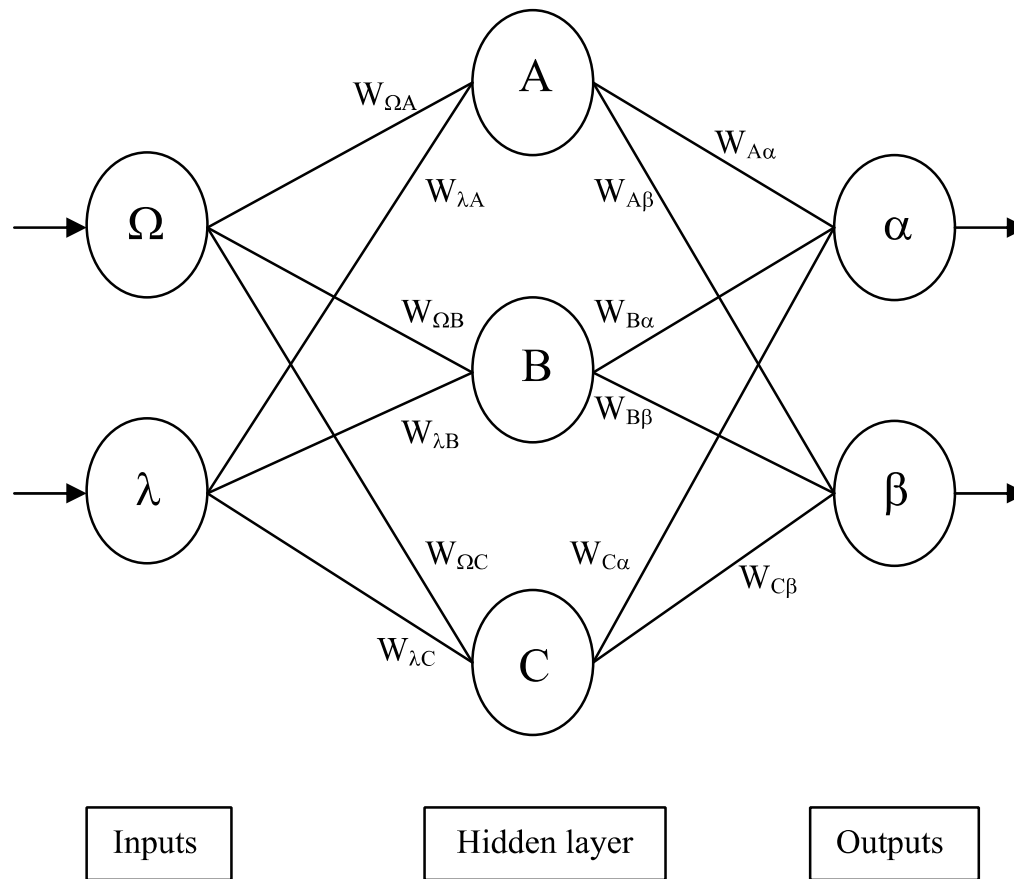
hidden
neuron:

$$w_{j,m}^{(1)} = \Delta w_{j,m}^{(1)} + w_{j,m}^{(1)} \leftarrow \frac{\Delta w_{j,m}^{(1)}}{\Delta w_{j,m}^{(1)}} = \eta \text{Error}_j \text{Output}_m = \eta \delta_j x_m \quad E(W) = \frac{1}{2} \sum_k (y_k - d_k)^2$$

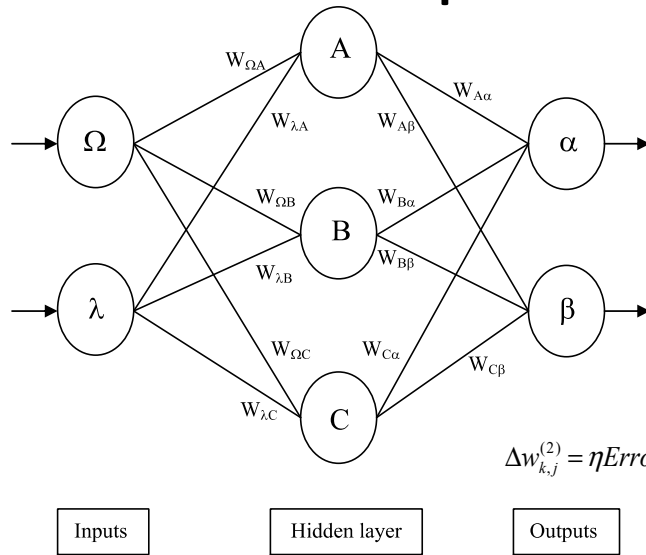
$$\Delta w_{j,m}^{(1)} = -\eta \frac{\partial E(W)}{\partial w_{j,m}^{(1)}} = -\eta \frac{\partial E(W)}{\partial h_j^{(1)}} \frac{\partial h_j^{(1)}}{\partial w_{j,m}^{(1)}} = \eta \left[\sum_k (d_k - y_k) f_{(2)}'(net_k^{(2)}) w_{k,j}^{(2)} \right] [x_m f_{(1)}'(net_j^{(1)})] = \eta \delta_j x_m$$

$$\text{Error: } \delta_j \equiv f_{(1)}'(net_j^{(1)}) \sum_k (d_k - y_k) f_{(2)}'(net_k^{(2)}) w_{k,j}^{(2)} = f_{(1)}'(net_j^{(1)}) \sum_k \delta_k w_{k,j}^{(2)}$$

An example for Backprop



An example for Backprop



1. Calculate errors of output neurons

$$\delta_\alpha = \text{out}_\alpha (1 - \text{out}_\alpha) (\text{Target}_\alpha - \text{out}_\alpha)$$

$$\delta_\beta = \text{out}_\beta (1 - \text{out}_\beta) (\text{Target}_\beta - \text{out}_\beta)$$

$$\delta_k = (d_k - y_k) f'_{(2)}(\text{net}_k^{(2)})$$

2. Change output layer weights

$$\begin{aligned} W_{A\alpha}^+ &= W_{A\alpha} + \eta \delta_\alpha \text{out}_A & W_{A\beta}^+ &= W_{A\beta} + \eta \delta_\beta \text{out}_A \\ W_{B\alpha}^+ &= W_{B\alpha} + \eta \delta_\alpha \text{out}_B & W_{B\beta}^+ &= W_{B\beta} + \eta \delta_\beta \text{out}_B \\ W_{C\alpha}^+ &= W_{C\alpha} + \eta \delta_\alpha \text{out}_C & W_{C\beta}^+ &= W_{C\beta} + \eta \delta_\beta \text{out}_C \end{aligned}$$

$$\Delta w_{k,j}^{(2)} = \eta \text{Error}_k \text{Output}_j = \eta \delta_k h_j^{(1)}$$

3. Calculate (back-propagate) hidden layer errors

$$\begin{aligned} \delta_A &= \text{out}_A (1 - \text{out}_A) (\delta_\alpha W_{A\alpha} + \delta_\beta W_{A\beta}) \\ \delta_B &= \text{out}_B (1 - \text{out}_B) (\delta_\alpha W_{B\alpha} + \delta_\beta W_{B\beta}) \\ \delta_C &= \text{out}_C (1 - \text{out}_C) (\delta_\alpha W_{C\alpha} + \delta_\beta W_{C\beta}) \end{aligned}$$

$$\delta_j = f'_{(1)}(\text{net}_j^{(1)}) \sum_k \delta_k w_{k,j}^{(2)}$$

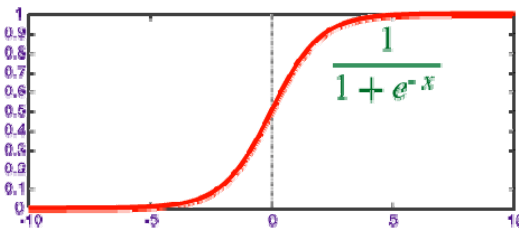
4. Change hidden layer weights

$$\begin{aligned} W_{\lambda A}^+ &= W_{\lambda A} + \eta \delta_A \text{in}_\lambda & W_{\Omega A}^+ &= W_{\Omega A} + \eta \delta_A \text{in}_\Omega \\ W_{\lambda B}^+ &= W_{\lambda B} + \eta \delta_B \text{in}_\lambda & W_{\Omega B}^+ &= W_{\Omega B} + \eta \delta_B \text{in}_\Omega \\ W_{\lambda C}^+ &= W_{\lambda C} + \eta \delta_C \text{in}_\lambda & W_{\Omega C}^+ &= W_{\Omega C} + \eta \delta_C \text{in}_\Omega \end{aligned}$$

$$\Delta w_{j,m}^{(1)} = \eta \text{Error}_j \text{Output}_m = \eta \delta_j x_m$$

Consider sigmoid activation function

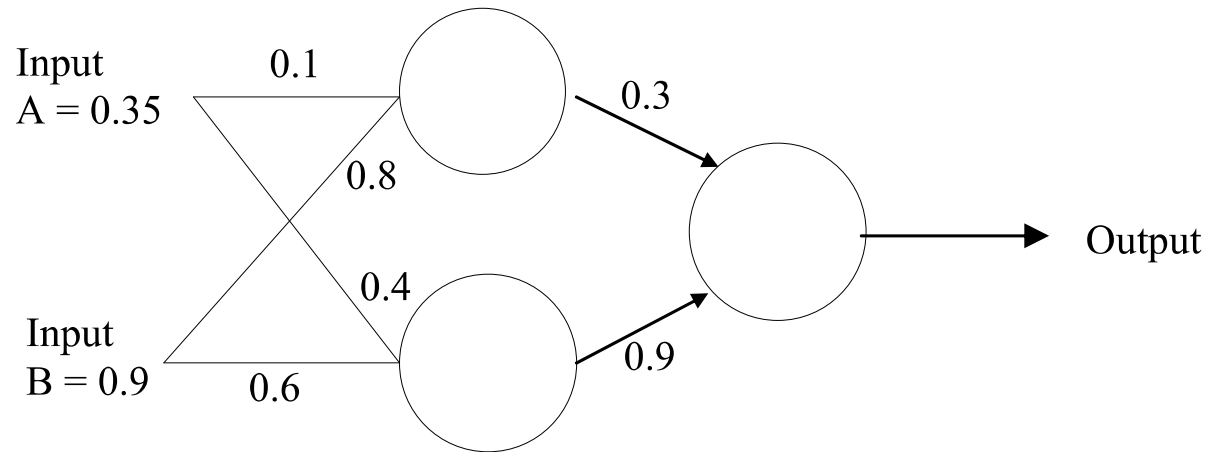
$$f_{\text{Sigmoid}}(x) = \frac{1}{1 + e^{-x}}$$



$$f'_{\text{Sigmoid}}(x) = f_{\text{Sigmoid}}(x)(1 - f_{\text{Sigmoid}}(x))$$

Let us do some calculation

Consider the simple network below:



Assume that the neurons have a Sigmoid activation function and

- (i) Perform a forward pass on the network.
- (ii) Perform a reverse pass (training) once (target = 0.5).
- (iii) Perform a further forward pass and comment on the result.

Let us do some calculation

Answer:

(i)

Input to top neuron = $(0.35 \times 0.1) + (0.9 \times 0.8) = 0.755$. Out = 0.68.

Input to bottom neuron = $(0.9 \times 0.6) + (0.35 \times 0.4) = 0.68$. Out = 0.6637.

Input to final neuron = $(0.3 \times 0.68) + (0.9 \times 0.6637) = 0.80133$. Out = 0.69.

(ii)

Output error $\delta = (t - o)(1 - o)o = (0.5 - 0.69)(1 - 0.69)0.69 = -0.0406$.

New weights for output layer

$w1^+ = w1 + (\delta \times \text{input}) = 0.3 + (-0.0406 \times 0.68) = 0.272392$.

$w2^+ = w2 + (\delta \times \text{input}) = 0.9 + (-0.0406 \times 0.6637) = 0.87305$.

Errors for hidden layers:

$\delta1 = \delta \times w1 = -0.0406 \times 0.272392 \times (1 - o)o = -2.406 \times 10^{-3}$

$\delta2 = \delta \times w2 = -0.0406 \times 0.87305 \times (1 - o)o = -7.916 \times 10^{-3}$

New hidden layer weights:

$w3^+ = 0.1 + (-2.406 \times 10^{-3} \times 0.35) = 0.09916$.

$w4^+ = 0.8 + (-2.406 \times 10^{-3} \times 0.9) = 0.7978$.

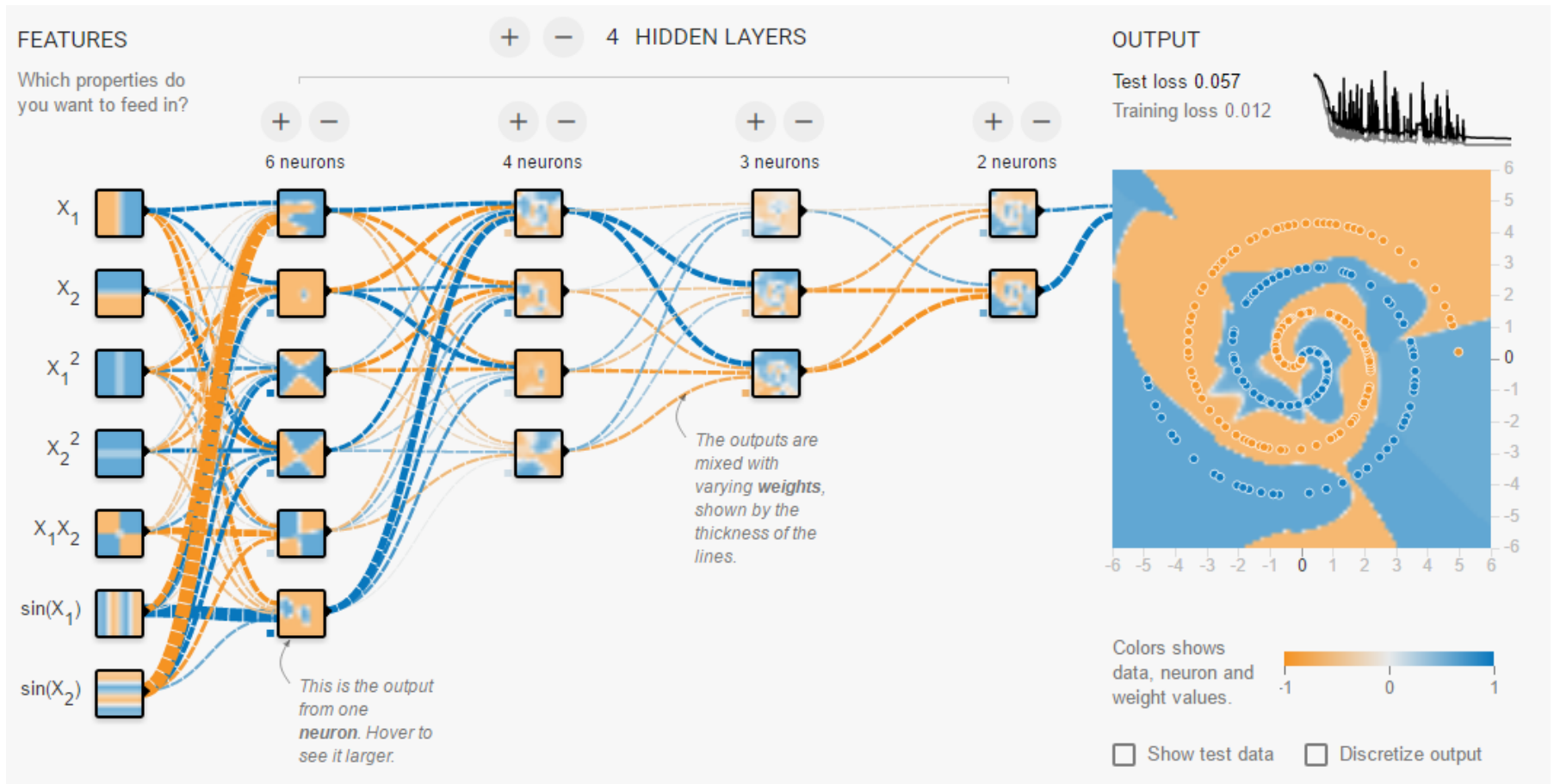
$w5^+ = 0.4 + (-7.916 \times 10^{-3} \times 0.35) = 0.3972$.

$w6^+ = 0.6 + (-7.916 \times 10^{-3} \times 0.9) = 0.5928$.

(iii)

Old error was -0.19. New error is -0.18205. Therefore error has reduced.

A demo from Google

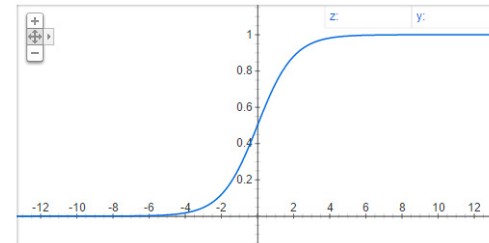


<http://playground.tensorflow.org/>

Non-linear Activation Functions

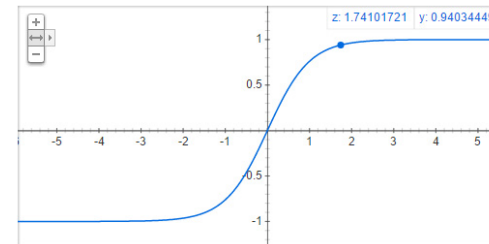
- Sigmoid

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



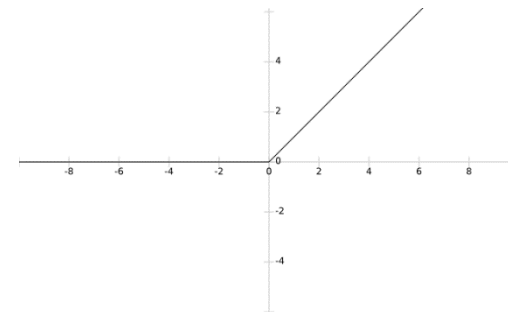
- Tanh

$$\tanh(z) = \frac{1 - e^{-2z}}{1 + e^{-2z}}$$



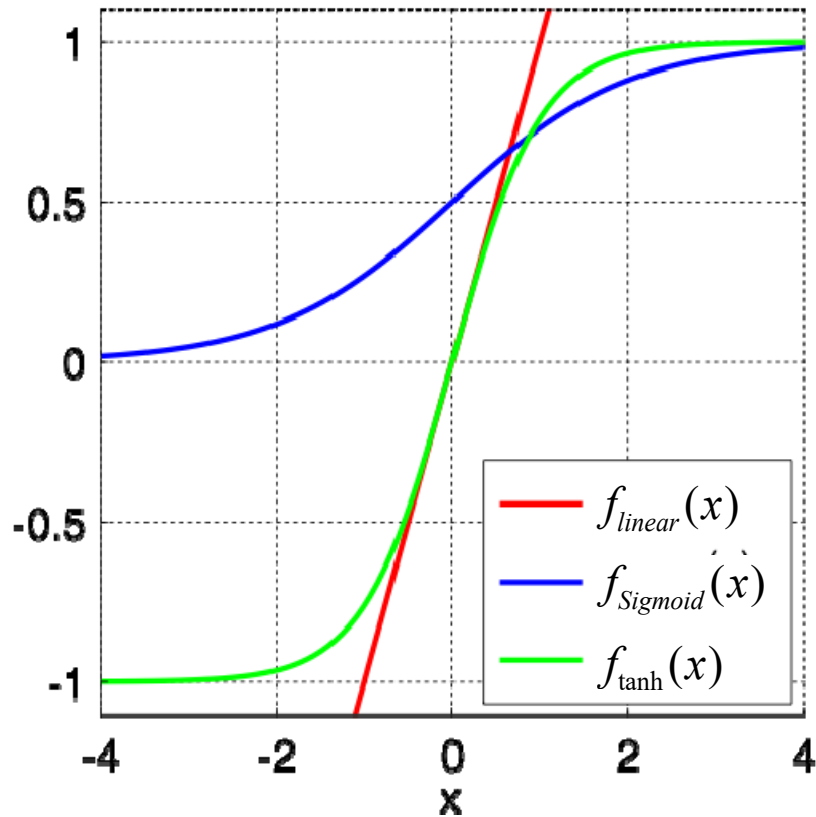
- Rectified Linear Unit (ReLU)

$$\text{ReLU}(z) = \max(0, z)$$

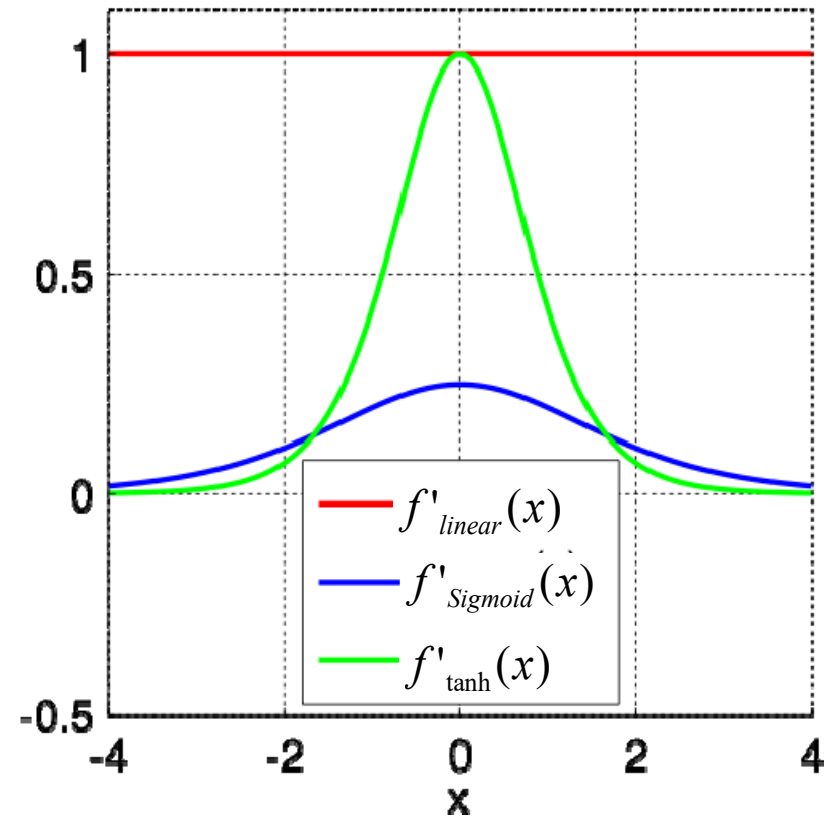


Active functions

Some Common Activation Functions



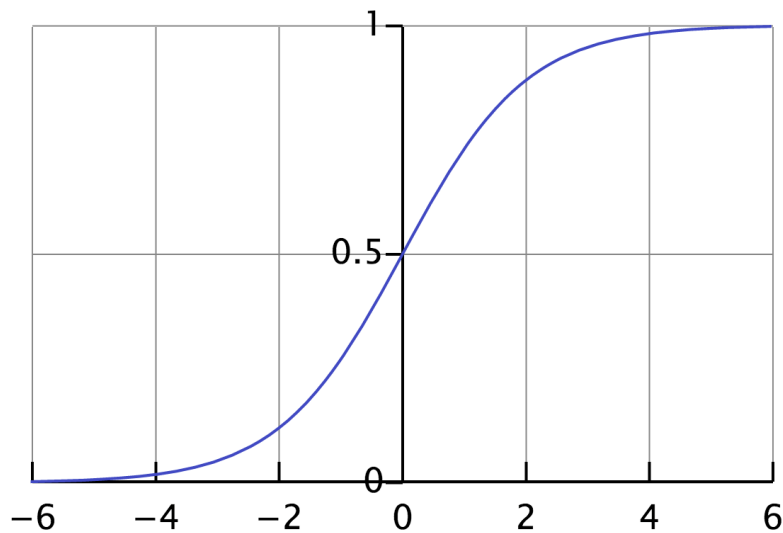
Activation Function Derivatives



Activation functions

- **Logistic Sigmoid:**

$$f_{Sigmoid}(x) = \frac{1}{1 + e^{-x}}$$



Its derivative:

$$f'_{Sigmoid}(x) = f_{Sigmoid}(x)(1 - f_{Sigmoid}(x))$$

- Output range [0,1]
- Motivated by biological neurons and can be interpreted as the probability of an artificial neuron “firing” given its inputs
- However, saturated neurons make gradients vanished (**why?**)

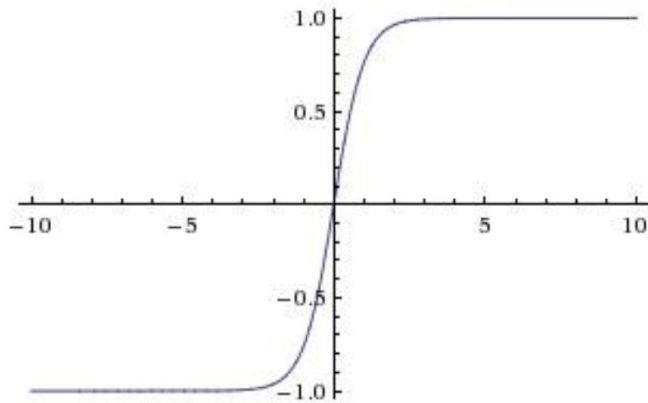
Activation functions

- Tanh function

$$f_{\tanh}(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Its gradient:

$$f_{\tanh}(x) = 1 - f_{\tanh}(x)^2$$

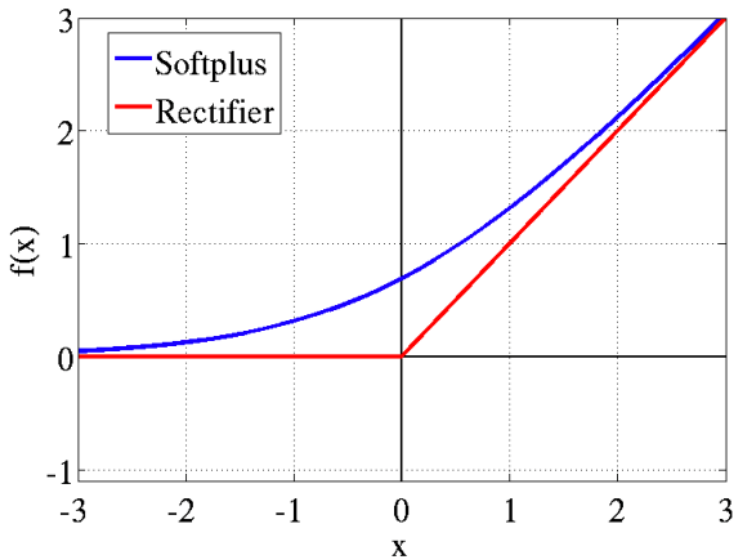


- Output range [-1,1]
- Thus strongly negative inputs to the tanh will map to negative outputs.
- Only zero-valued inputs are mapped to near-zero outputs
- These properties make the network less likely to get “stuck” during training

Active Functions

- ReLU (rectified linear unit)

$$f_{\text{ReLU}}(x) = \max(0, x)$$



<http://static.googleusercontent.com/media/research.google.com/en//pubs/archive/40811.pdf>

- The derivative:

$$f_{\text{ReLU}}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

- Another version is

Noise ReLU:

$$f_{\text{NoisyReLU}}(x) = \max(0, x + N(0, \delta(x)))$$

- ReLU can be approximated by **softplus function**

$$f_{\text{Softplus}}(x) = \log(1 + e^x)$$

- ReLU gradient doesn't vanish as we increase x
- It can be used to model positive number
- It is fast as no need for computing the exponential function
- It eliminates the necessity to have a **"pretraining"** phase

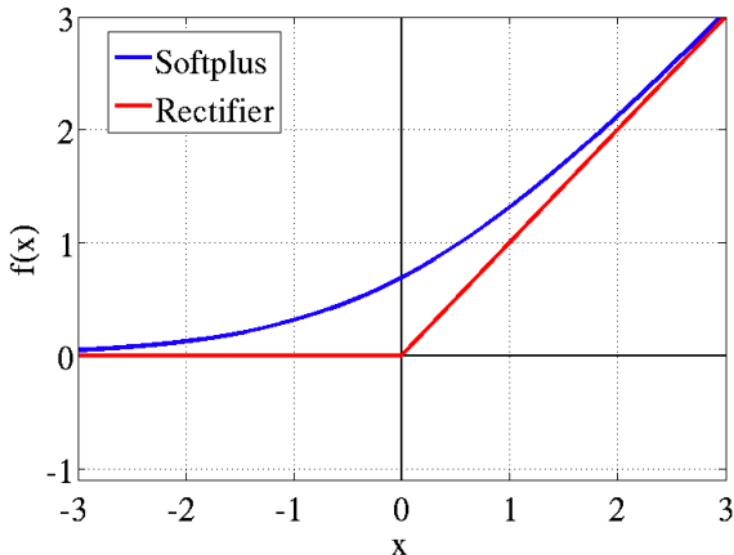
Active Functions

- **ReLU (rectified linear unit)**

$$f_{\text{ReLU}}(x) = \max(0, x)$$

ReLU can be approximated by **softplus function**

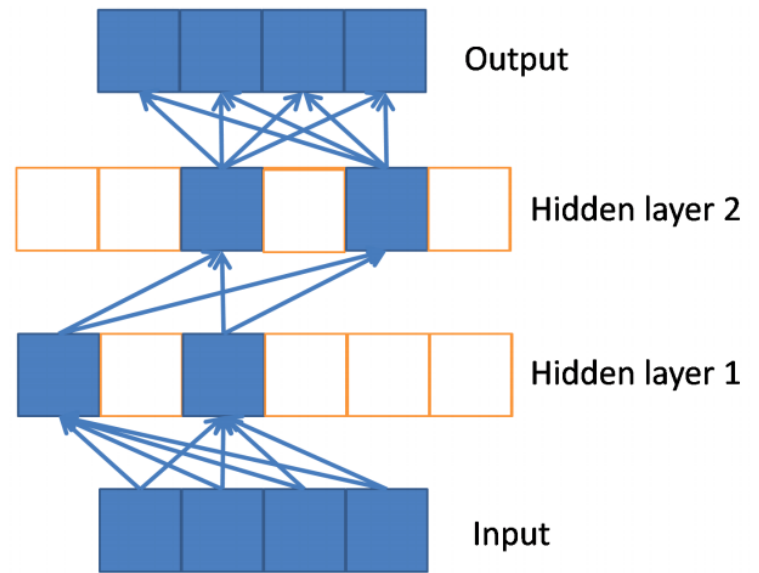
$$f_{\text{Softplus}}(x) = \log(1 + e^x)$$



Additional active functions:

Leaky ReLU, Exponential LU, Maxout etc

- The only non-linearity comes from the path selection with individual neurons being active or not
- It allows **sparse representations**:
 - for a given input only a subset of neurons are active



Sparse propagation of activations and gradients

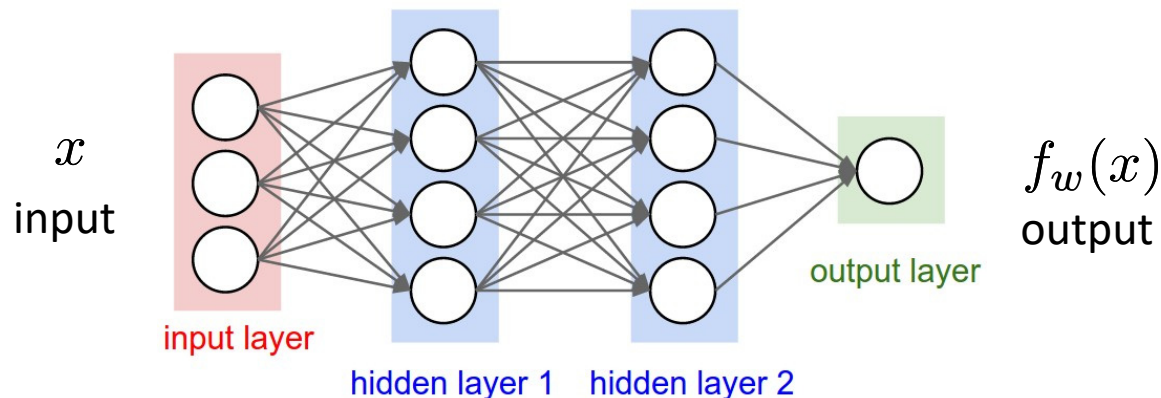
Error/Loss function

- Recall stochastic gradient descent
 - Update from a randomly picked example (but in practice do a batch update)

$$w = w - \eta \frac{\partial \mathcal{L}(w)}{\partial w}$$

- Squared error loss for one binary output:

$$\mathcal{L}(w) = \frac{1}{2}(y - f_w(x))^2$$



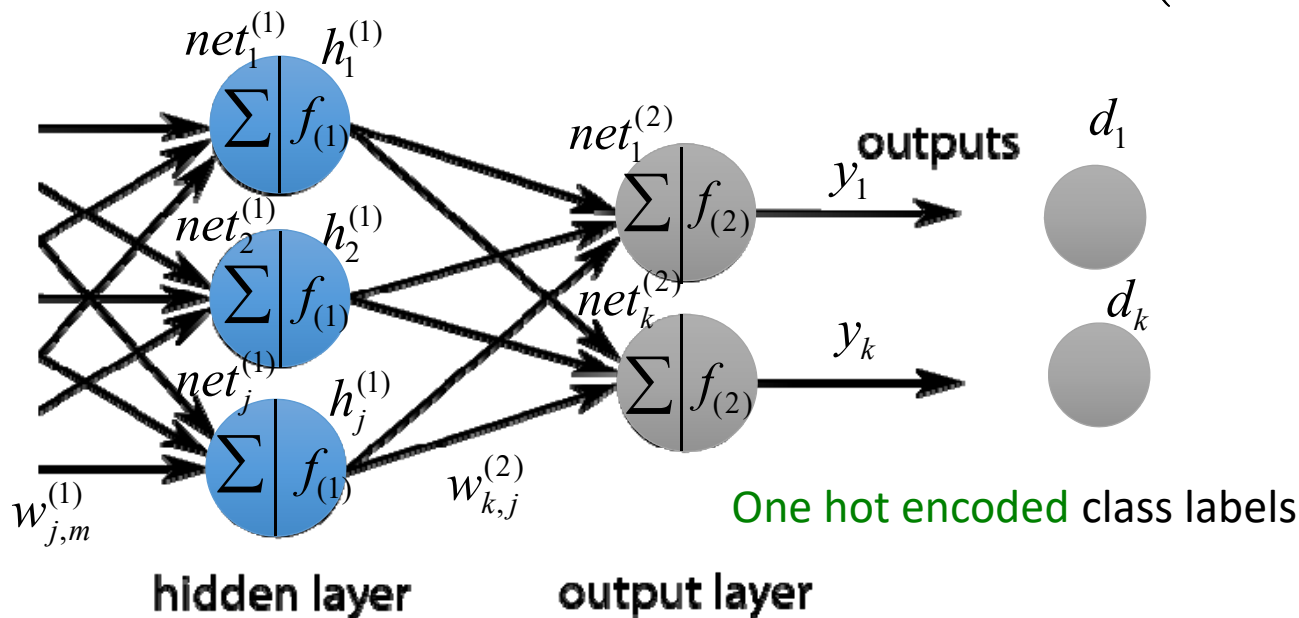
Error/Loss function

- **Softmax** (cross-entropy loss) for multiple classes

(Class labels follow multinomial distribution)

$$\mathcal{L}(w) = - \sum_k (d_k \log \hat{y}_k + (1 - d_k) \log(1 - y_k))$$

where $\hat{y}_k = \frac{\exp \left(\sum_j w_{k,j}^{(2)} h_j^{(1)} \right)}{\sum_{k'} \exp \left(\sum_j w_{k',j}^{(2)} h_j^{(1)} \right)}$



Advanced Topic of this Lecture

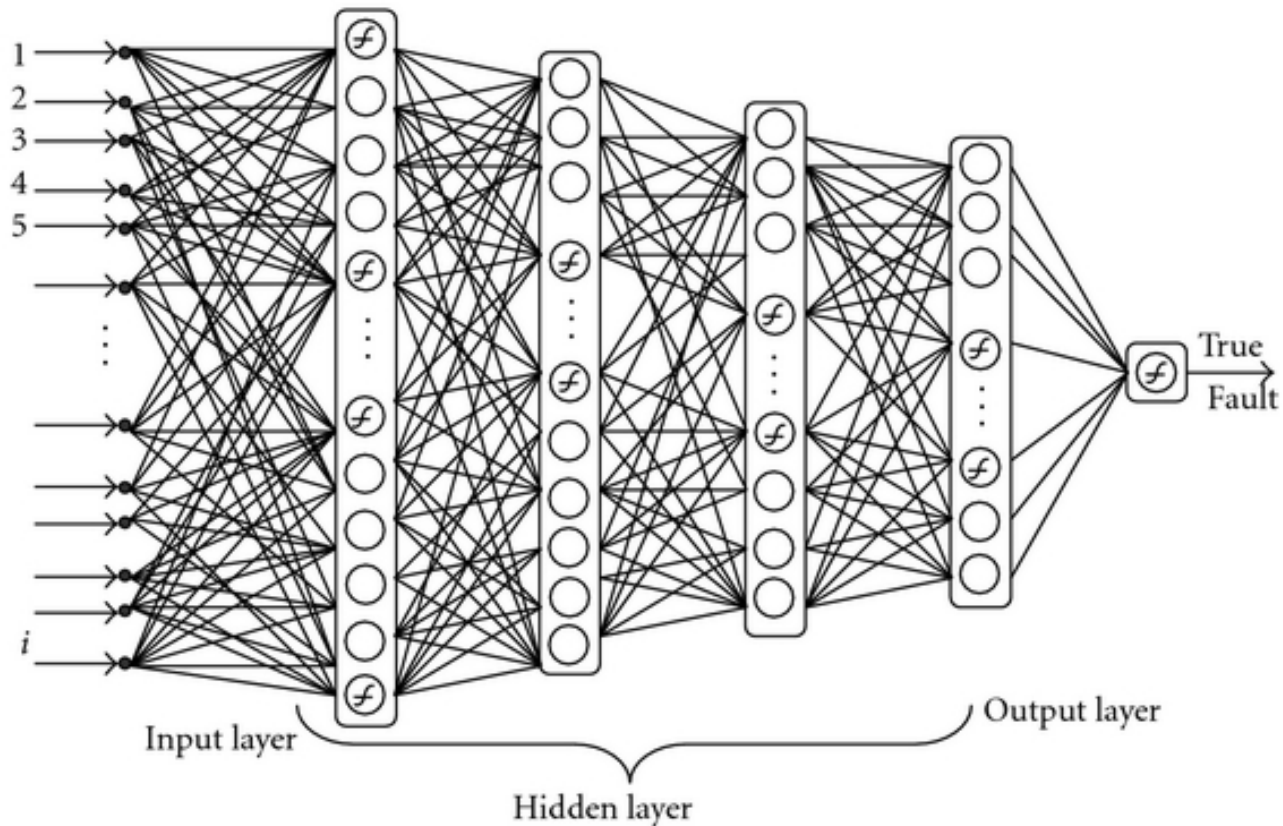
Deep Learning

As a prologue of the DL Course in the next semester

What is Deep Learning

- Deep learning methods are representation-learning methods with **multiple levels of representation**, obtained by composing simple but **non-linear modules** that each transform the representation at one level (starting with the raw input) into a representation at a higher, slightly more abstract level.
- Mostly implemented via neural networks

Deep Neural Network (DNN)



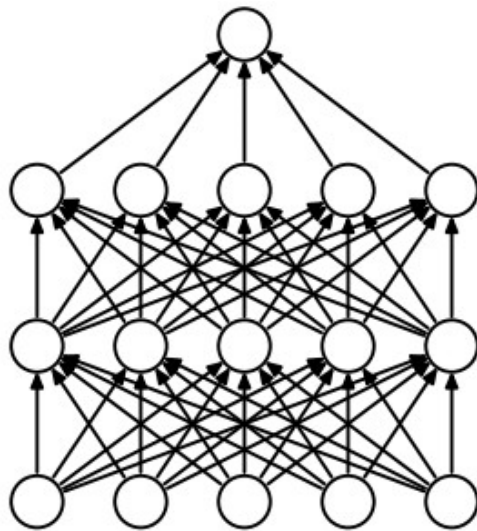
- Multi-layer perceptron with many hidden layers

Difficulty of Training Deep Nets

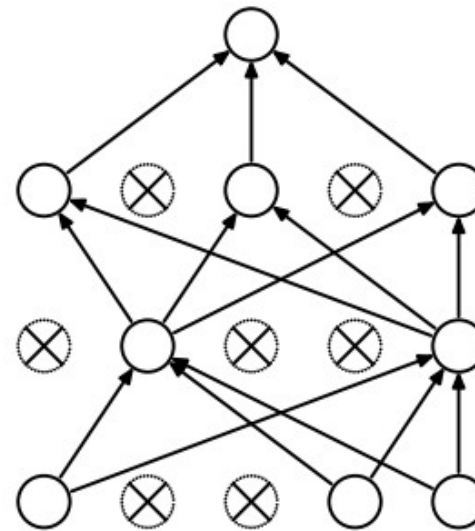
- Lack of big data
 - Now we have a lot of big data
- Lack of computational resources
 - Now we have GPUs and HPCs
- Easy to get into a (bad) local minimum
 - Now we use pre-training techniques, various optimization algorithms
- Gradient vanishing
 - Now we use ReLU
- Regularization
 - Now we use Dropout

Dropout

- Dropout randomly 'drops' units from a layer on each training step, creating 'sub-architectures' within the model.
- It can be viewed as a type of sampling of a smaller network within a larger network
- Prevent Neural Networks from Overfitting



(a) Standard Neural Net

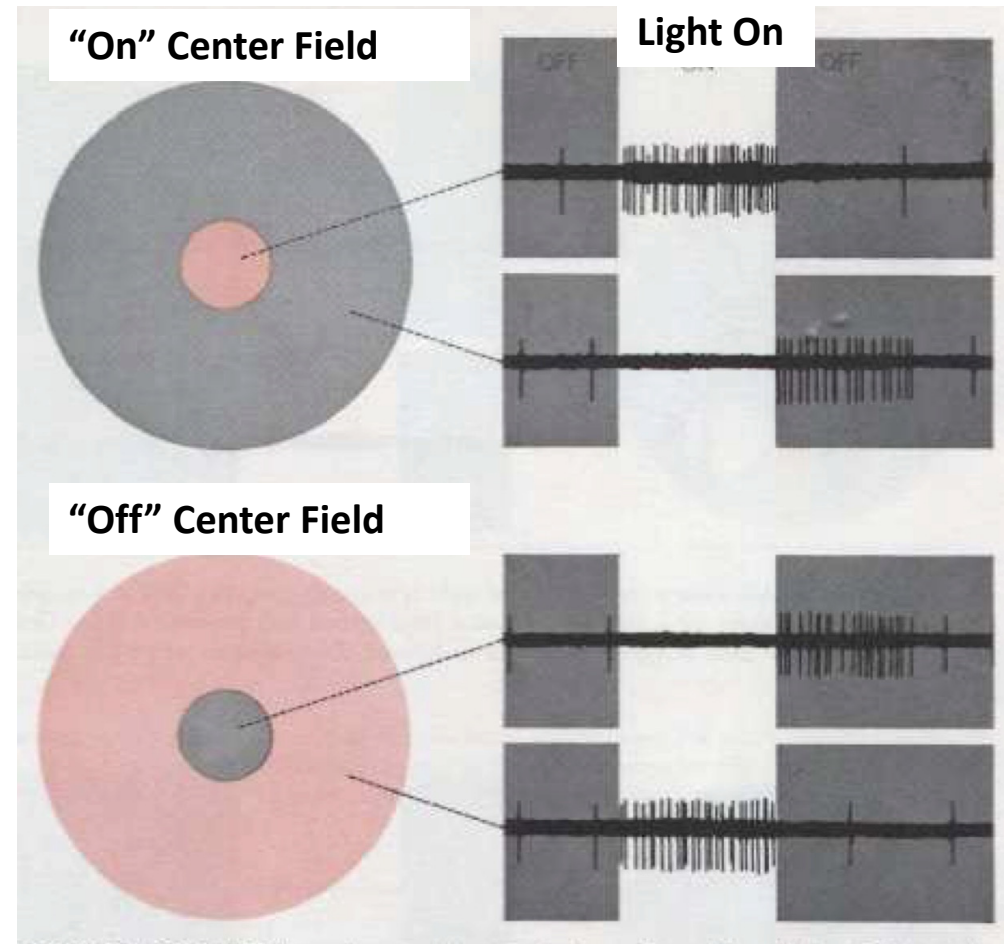


(b) After applying dropout.

Srivastava, Nitish, et al. "Dropout: A simple way to prevent neural networks from overfitting." The Journal of Machine Learning Research 15.1 (2014): 1929-1958.

Convolutional neural networks: Receptive field

- **Receptive field:** Neurons in the retina respond to light stimulus in restricted regions of the visual field
- Animal experiments on receptive fields of two retinal ganglion cells
 - Fields are circular areas of the retina
 - The cell (upper part) responds when the center is illuminated and the surround is darkened.
 - The cell (lower part) responds when the center is darkened and the surround is illuminated.
 - Both cells give on- and off-responses when both center and surround are illuminated, but neither response is as strong as when only center or surround is illuminated



Convolutional neural networks

- **Sparse connectivity** by local correlation
 - Filter: the input of a hidden unit in layer m are from a subset of units in layer $m-1$ that have spatially connected **receptive fields**
- **Shared weights**
 - each filter is replicated across the entire visual field. These replicated units share the same weights and form a feature map.

2-d case (subscripts are weights)

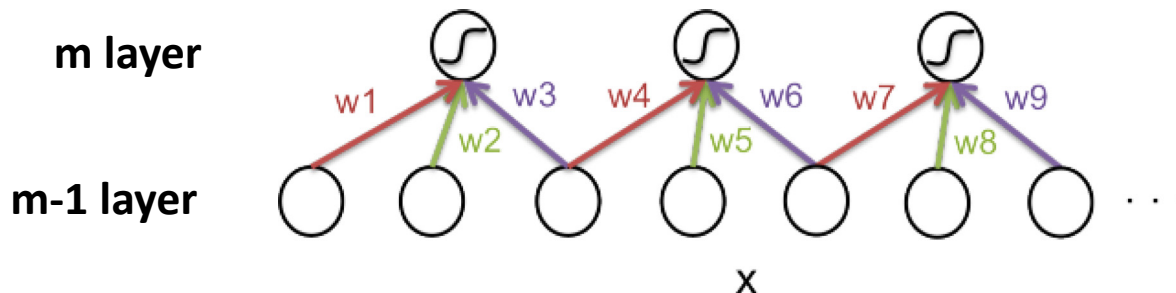
1 _{x1}	1 _{x0}	1 _{x1}	0	0
0 _{x0}	1 _{x1}	1 _{x0}	1	0
0 _{x1}	0 _{x0}	1 _{x1}	1	1
0	0	1	1	0
0	1	1	0	0

4		

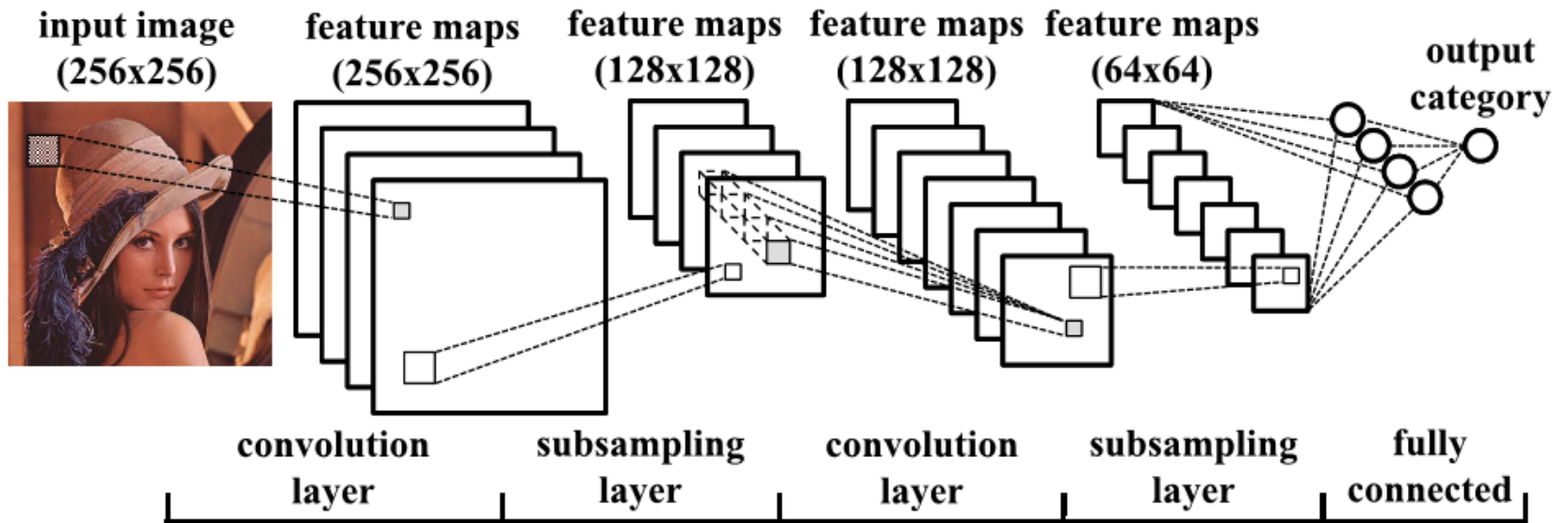
$m-1$ layer

one filter at m layer

1-d case

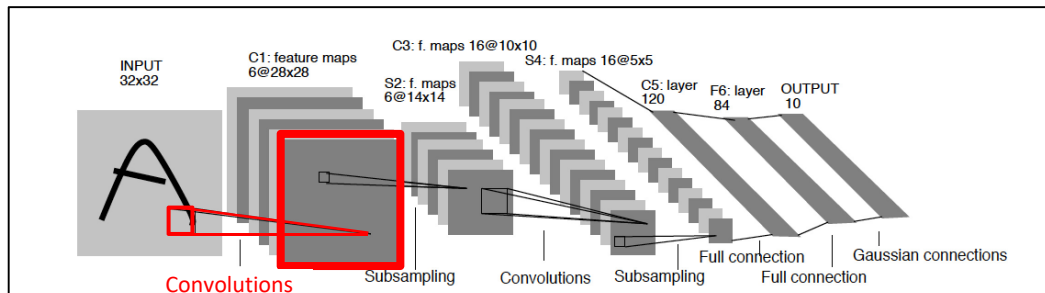
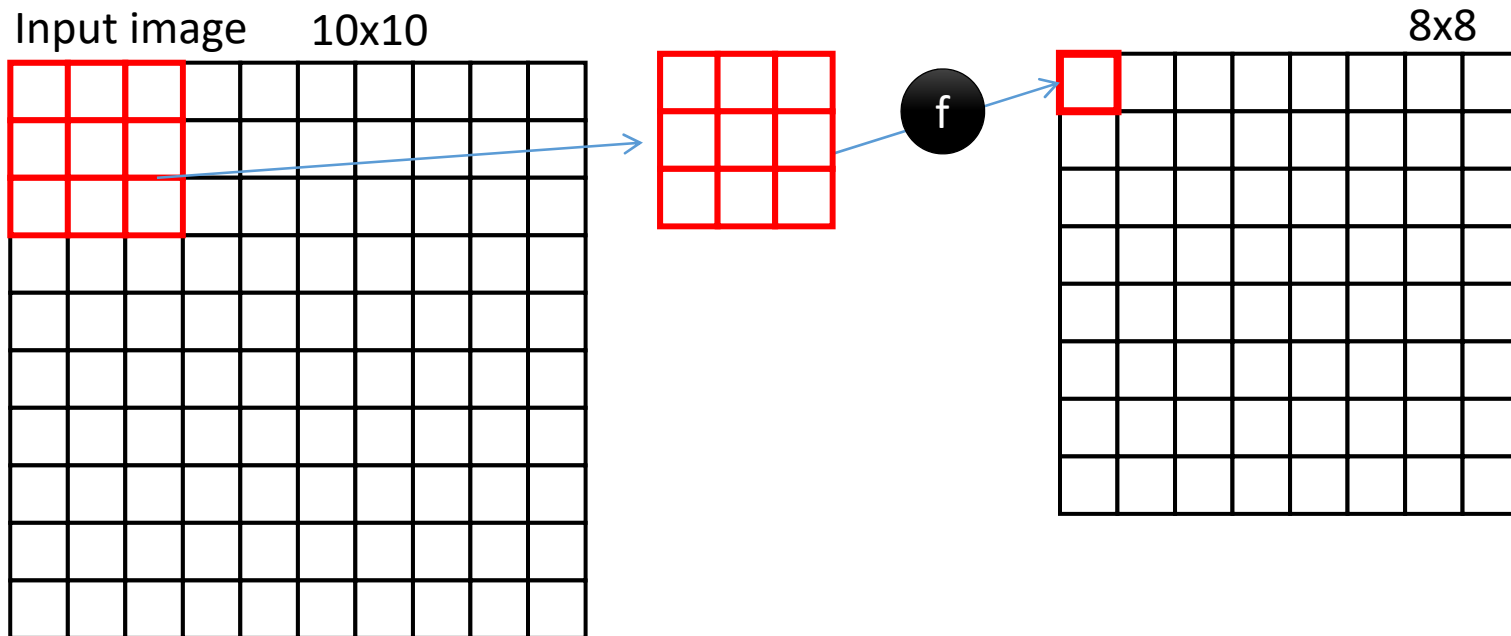


Convolutional Neural Network (CNN)



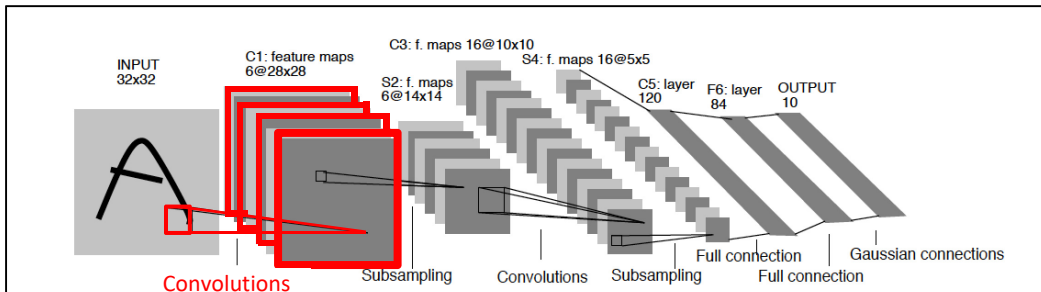
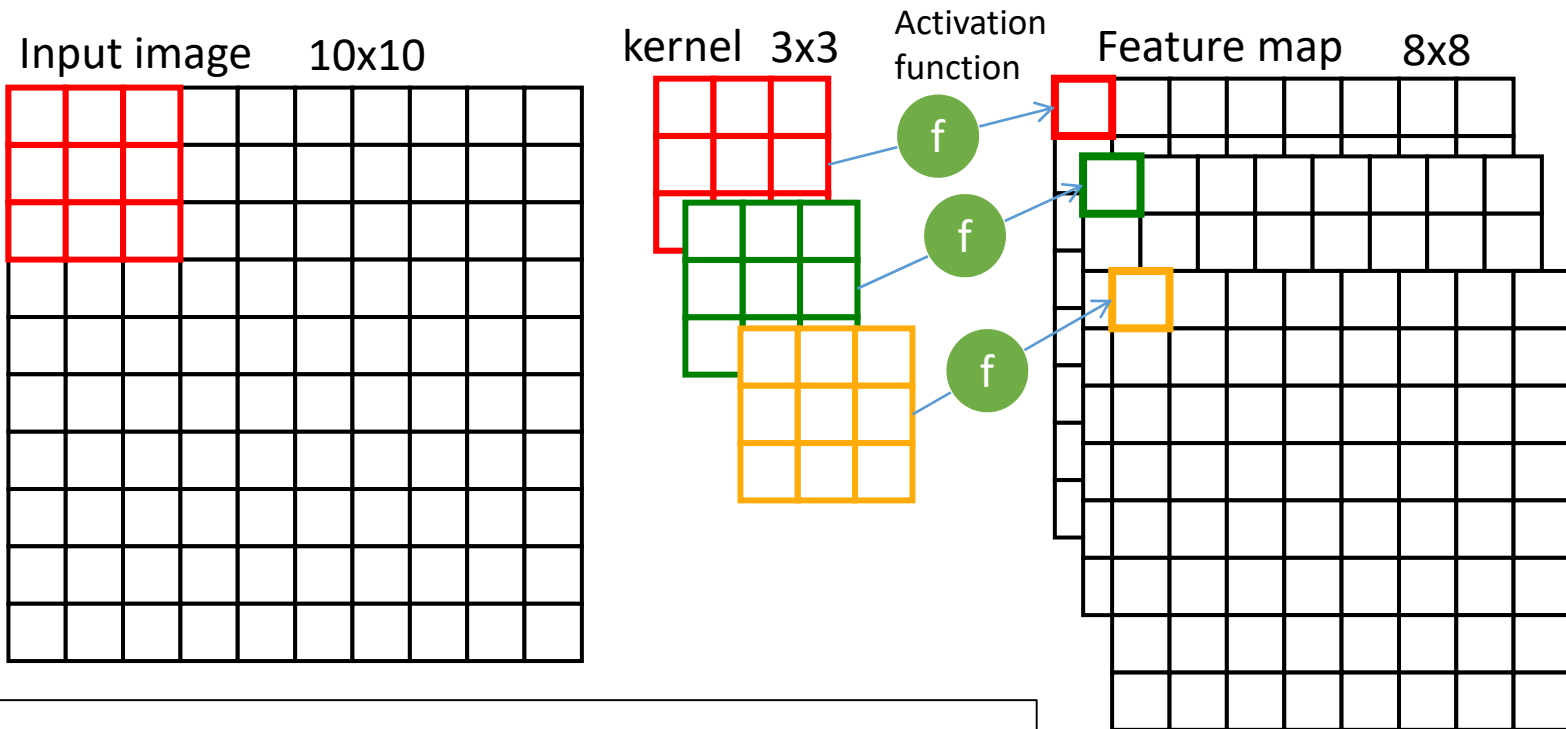
Convolution Layer

Example: a 10x10 input image with a 3x3 filter result in a 8x8 output image



Convolution Layer

- Example: a 10x10 input image with a 3x3 filter result in a 8x8 output image
- 3 different filters (weights are different) lead to 3 8x8 out images



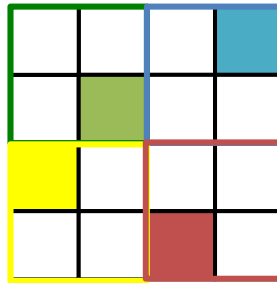
Pooling Subsampling Layer

- Pooling: partitions the input image into a set of non-overlapping rectangles and, for each such sub-region, outputs the maximum or average value.

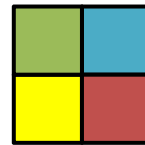
Max pooling

- reduces computation and
- is a way of taking the most responsive node of the given interest region,
- but may result in loss of accurate spatial information

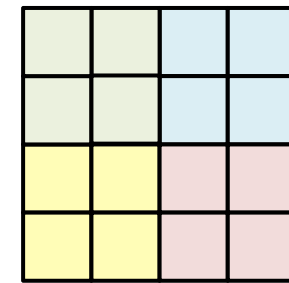
Max pooling



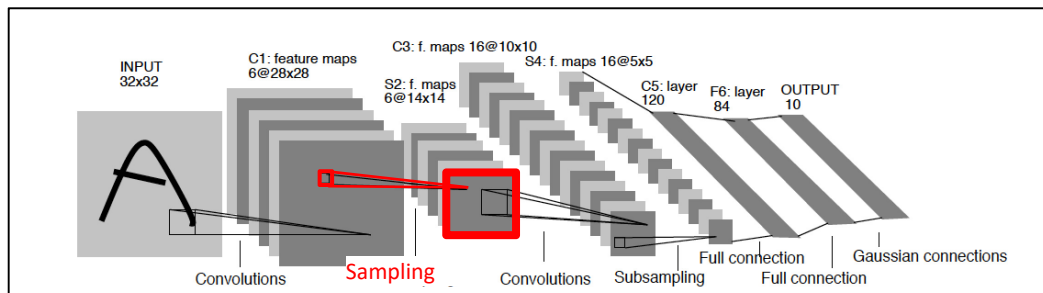
Max in a 2x2 filter



Average pooling



Average in a 2x2 filter



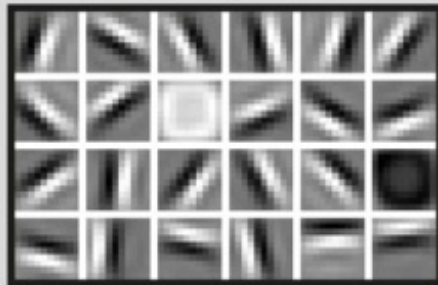
Use Case: Face Recognition

FACIAL RECOGNITION

Deep-learning neural networks use layers of increasingly complex rules to categorize complicated shapes such as faces.



Layer 1: The computer identifies pixels of light and dark.



Layer 2: The computer learns to identify edges and simple shapes.



Layer 3: The computer learns to identify more complex shapes and objects.



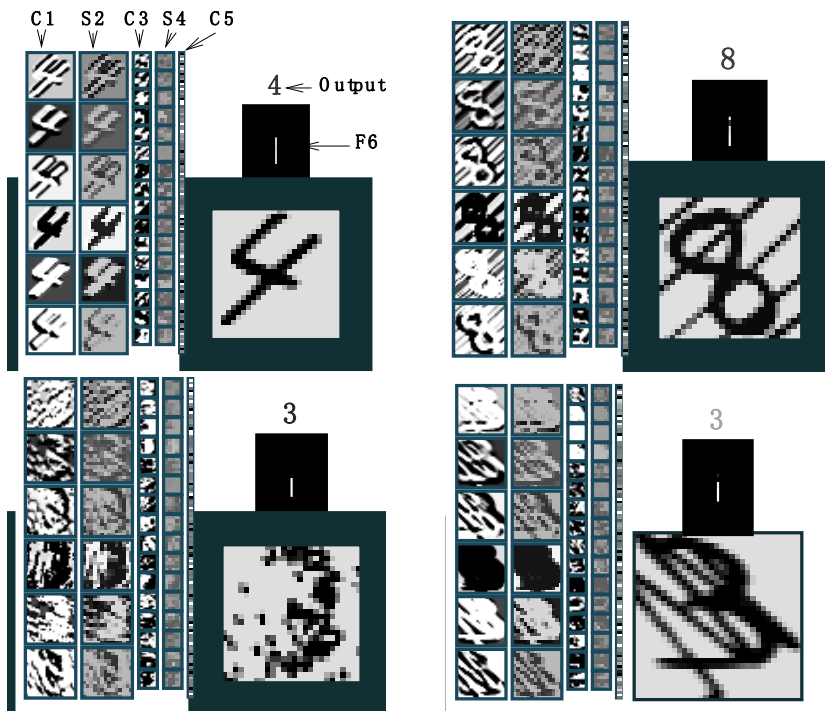
Layer 4: The computer learns which shapes and objects can be used to define a human face.

Use Case: Digits Recognition

- MNIST (handwritten digits) Dataset:

<http://yann.lecun.com/exdb/mnist/>

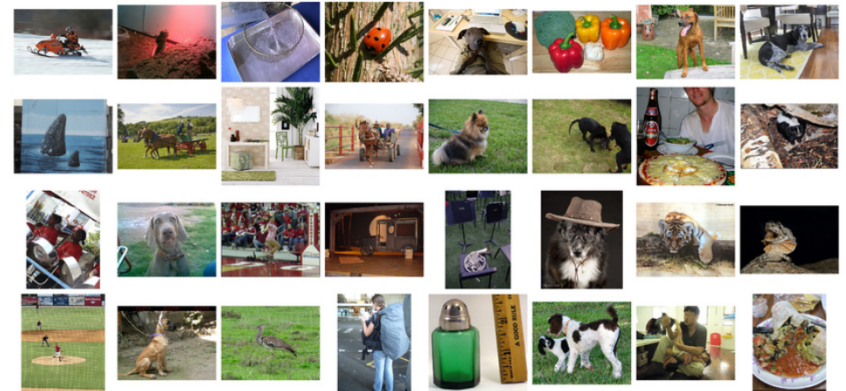
- 60k training and 10k test examples
- Test error rate **0.95%**



Total only 82 errors from LeNet-5. correct answer left and right is the machine answer.

More General Image Recognition

- ImageNet
 - Over 15M labeled high resolution images
 - Roughly 22K categories
 - Collected from web and labeled by Amazon Mechanical Turk
- The **Image/scene classification challenge**
 - Image/scene classification
 - Metric: **Hit@5 error rate** - make 5 guesses about the image label



Leadertable (ImageNet image classification)

2015 ResNet (ILSVRC'15) 3.57

Year	Codename	Error (percent)	99.9% Conf Int
2015	ResNet	3.57	
2014	GoogLeNet	6.66	6.40 - 6.92
2014	VGG	7.32	7.05 - 7.60
2014	MSRA	8.06	7.78 - 8.34
2014	AHoward	8.11	7.83 - 8.39
2014	DeeperVision	9.51	9.21 - 9.82
2013	Clarifai [†]	11.20	10.87 - 11.53
2014	CASIAWS [†]	11.36	11.03 - 11.69
2014	Trimps [†]	11.46	11.13 - 11.80
2014	Adobe [†]	11.58	11.25 - 11.91
2013	Clarifai	11.74	11.41 - 12.08
2013	NUS	12.95	12.60 - 13.30
2013	ZF	13.51	13.14 - 13.87
2013	AHoward	13.55	13.20 - 13.91
2013	OverFeat	14.18	13.83 - 14.54
2014	Orange [†]	14.80	14.43 - 15.17
2012	SuperVision [†]	15.32	14.94 - 15.69
2012	SuperVision	16.42	16.04 - 16.80
2012	ISI	26.17	25.71 - 26.65
2012	VGG	26.98	26.53 - 27.43
2012	XRCE	27.06	26.60 - 27.52
2012	UvA	29.58	29.09 - 30.04

Microsoft ResNet, a 152 layers network

GoogLeNet, 22 layers network

U. of Toronto, SuperVision, a 7 layers network

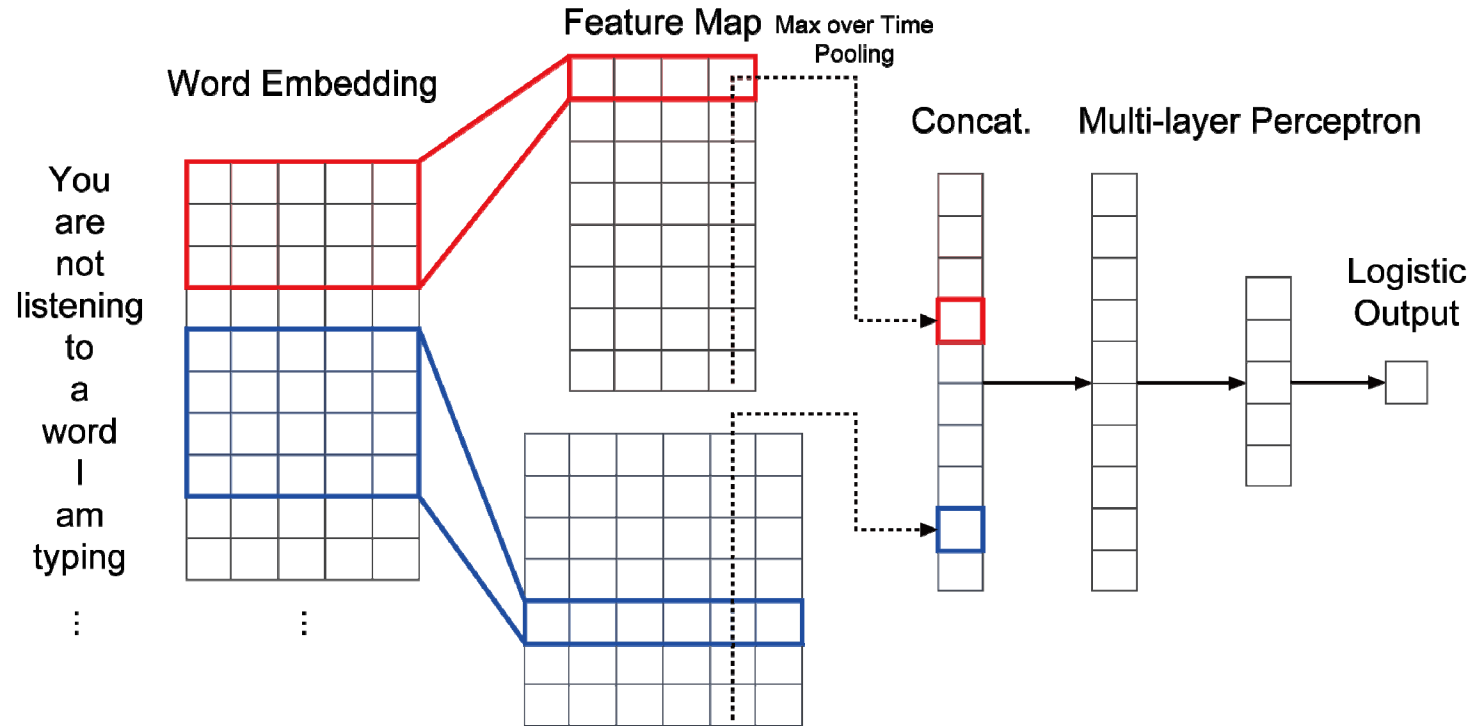
Unofficial human error is around 5.1% on a subset

Why human error still? When labeling, human raters judged whether it belongs to a class (binary classification); the challenge is a 1000-class classification problem.

<http://karpathy.github.io/2014/09/02/what-i-learned-from-competing-against-a-convnet-on-imagenet/>

Russakovsky O, Deng J, Su H, et al. Imagenet large scale visual recognition challenge[J]. International Journal of Computer Vision, 2015, 115(3): 211-252.

Use Case: Text Classification



- Word embedding: map each word to a k -dimensional dense vector
- CNN kernel: $n \times k$ matrix to explore the neighbor k words' patterns
- Max-over-time pooling: find the most salient pattern from the text for each kernel
- MLP: further feature interaction and distill high-level patterns

[Kim, Y. 2014. Convolutional neural networks for sentence classification. EMNLP 2014.]

Recurrent Neural Network (RNN)

- To model sequential data
 - Text
 - Time series
- Trained by Back-Propagation Through Time (BPTT)

x : input vector, o : output vector,

s : hidden state vector,

U : layer 1 param. matrix,

V : layer 2 param. matrix,

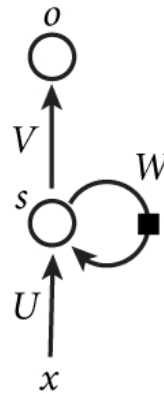
f : tanh or ReLU

$$o = f(sV)$$

$$s = f(xU)$$



Add time-dependency
of the hidden state s

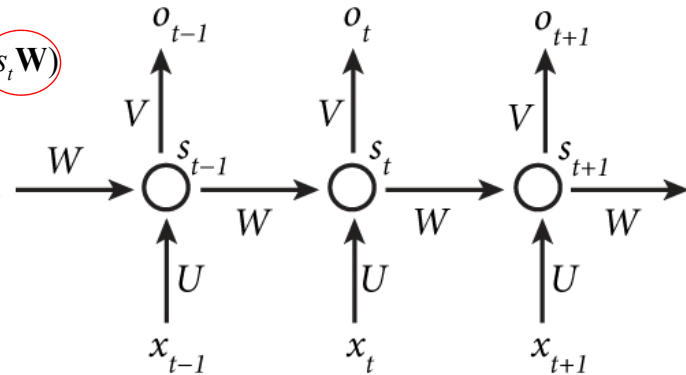


W : State transition param. matrix

$$o_{t+1} = f(s_{t+1} V)$$

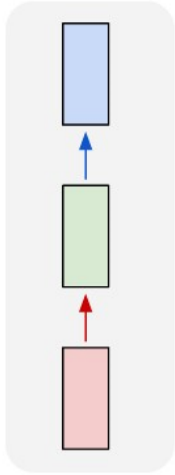
$$s_{t+1} = f(x_{t+1} U + s_t W)$$

Unfold



Different RNNs

one to one



Vanilla NN

one to many

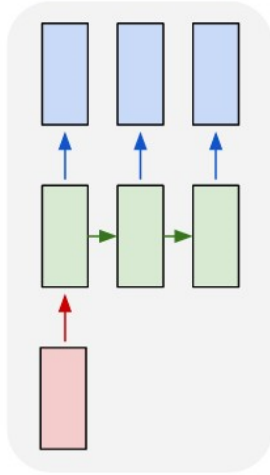
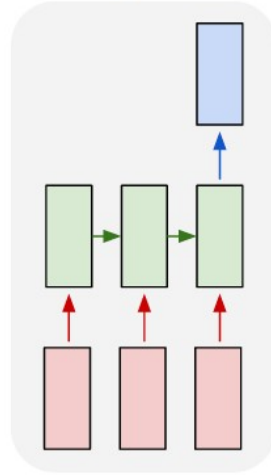


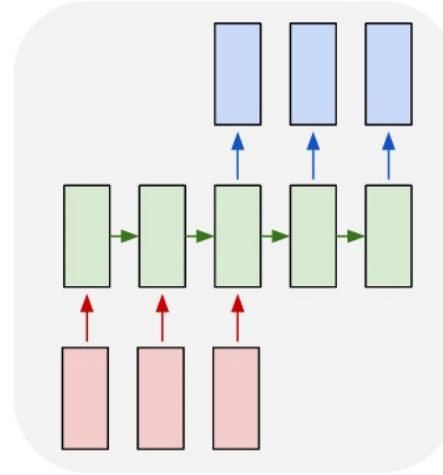
Image captioning
Text generation

many to one



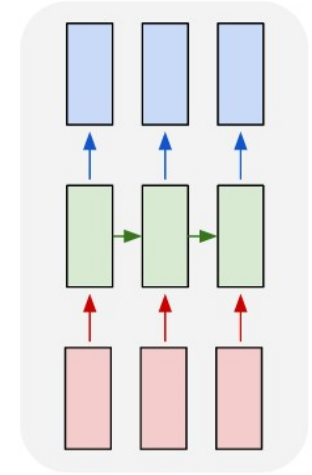
Text classification
Sentiment analysis

many to many



Machine translation
Dialogue system

many to many

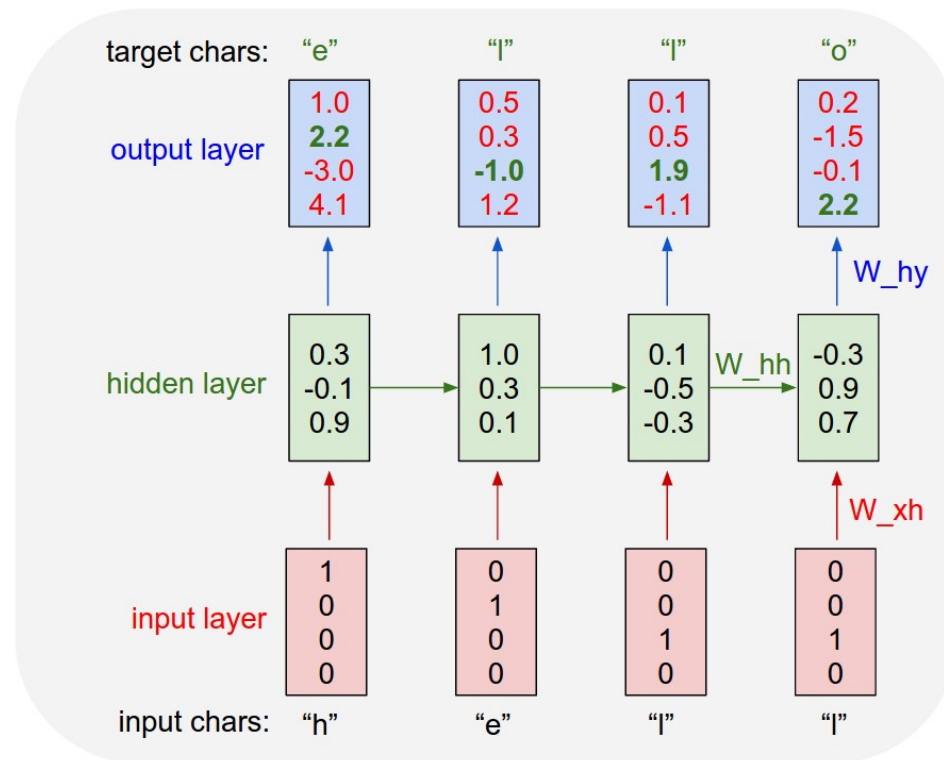


Stock price estimation
Video frame classification

- Different architecture for various tasks
- Strongly recommend Andrej Karpathy's blog
 - <http://karpathy.github.io/2015/05/21/rnn-effectiveness/>

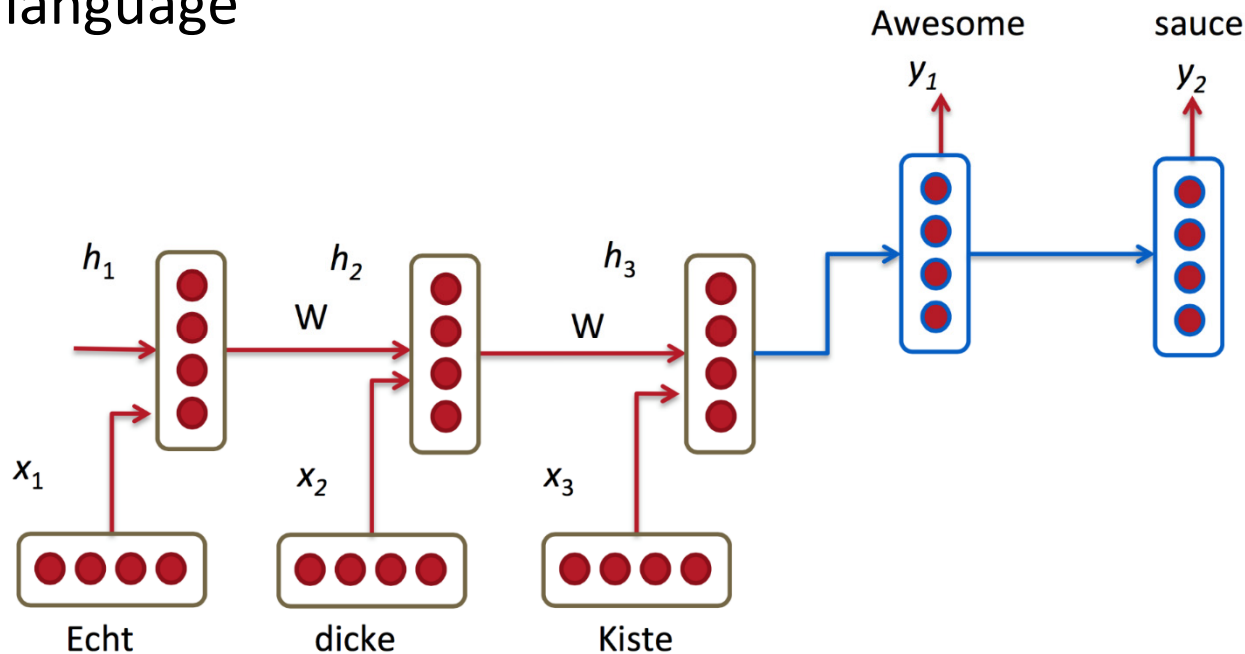
Use Case: Language Model

- Word-level or even character-level language model
 - Given previous words/characters, predict the next



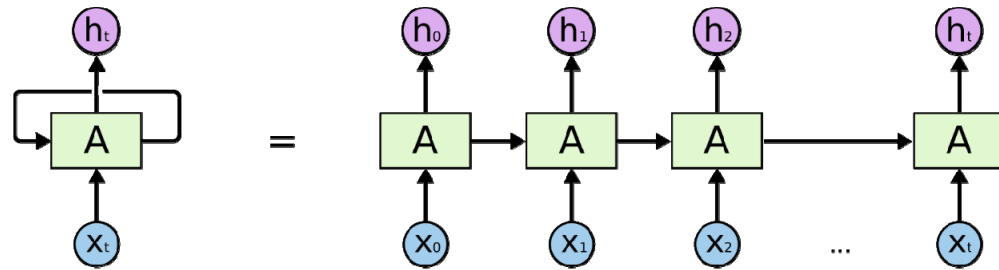
Use Case: Machine Translation

- Encode/decode RNN
 - First, encode the input sentence (into a vector e.g. h_3)
 - Then decode the vector into the sentence in another language

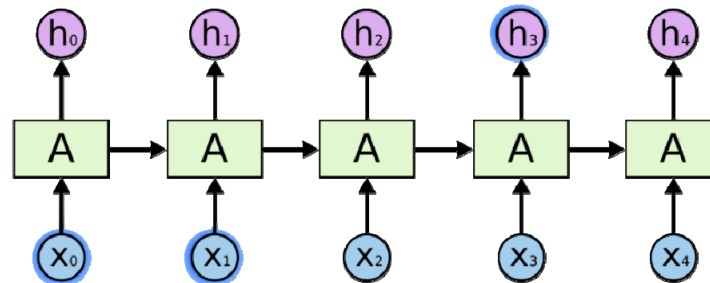


Problem of RNN

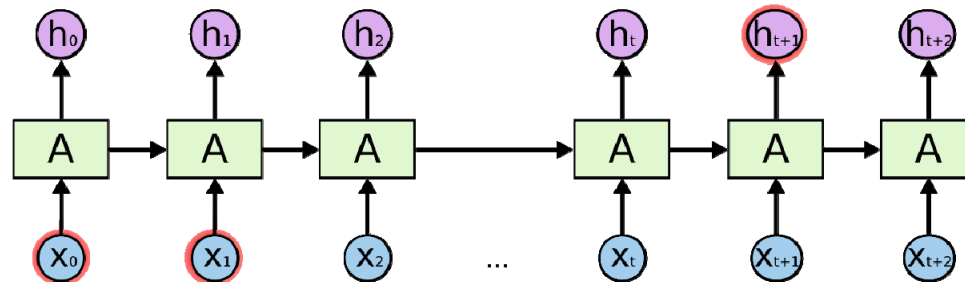
- Problem: RNN cannot nicely leverage the early information



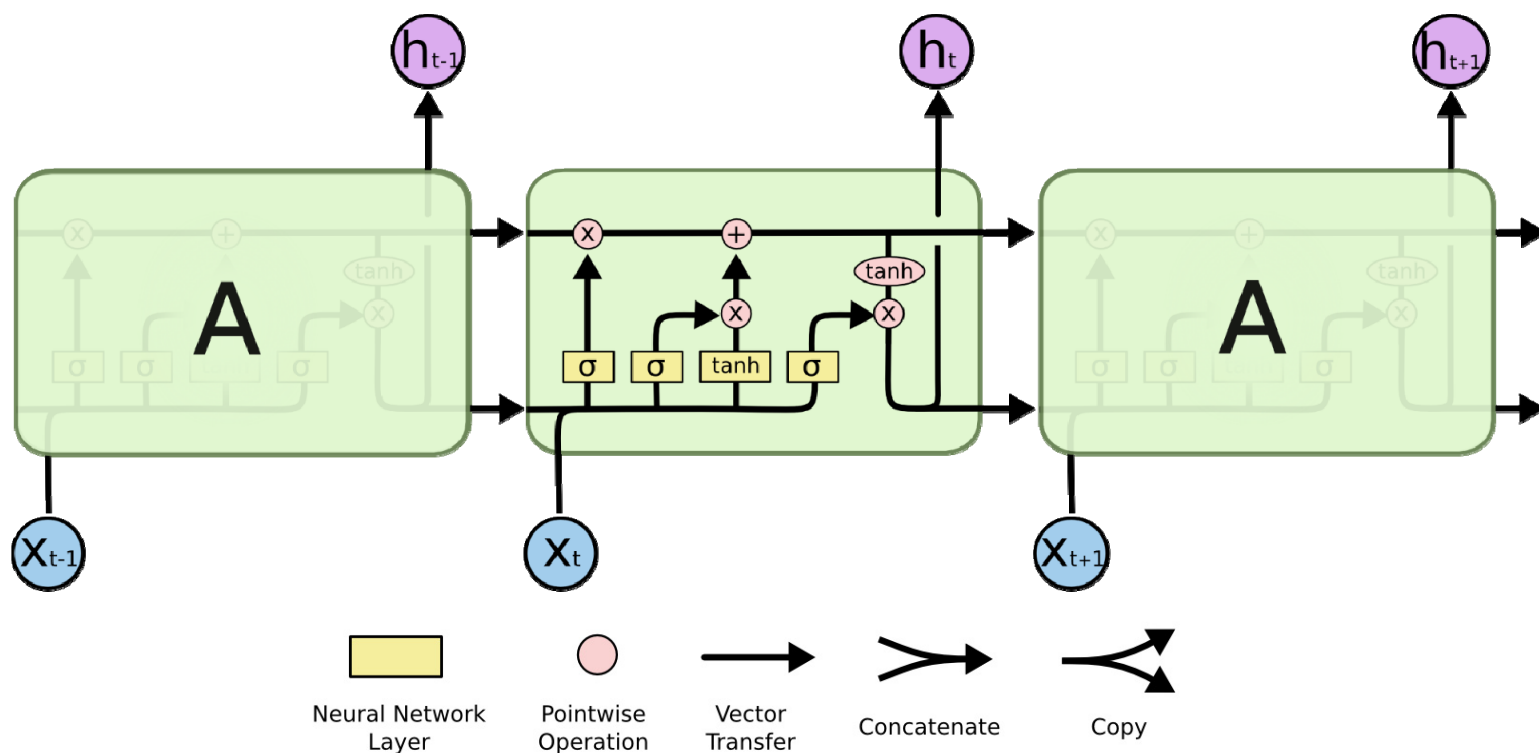
Gap dependency



Long-term dependency



Long Short-Term Memory (LSTM)

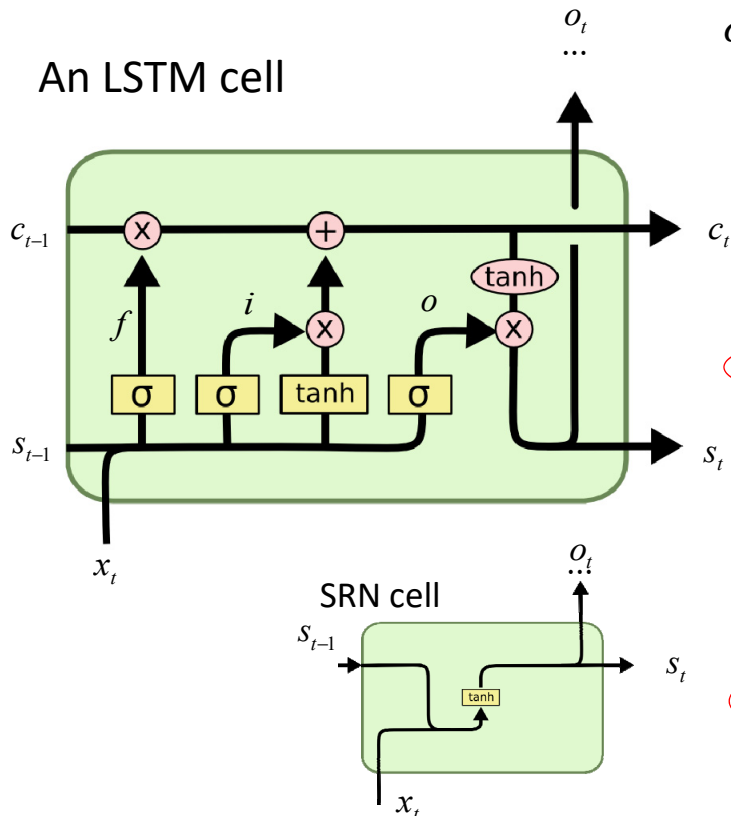


[<http://colah.github.io/posts/2015-08-Understanding-LSTMs/>]

[Hochreiter, Sepp, and Jürgen Schmidhuber. "Long short-term memory." Neural computation 9.8 (1997): 1735-1780.]

LSTM Cell

- An LSTM cell learn to decide which to remember/forget



σ : sigmoid (control signal between 0 and 1); \circ : elementwise multiplication

$$i = \sigma(x_t U^i + s_{t-1} W^i) \quad \text{input gate}$$

$$f = \sigma(x_t U^f + s_{t-1} W^f) \quad \text{forget gate}$$

$$o = \sigma(x_t U^o + s_{t-1} W^o) \quad \text{output gate}$$

$$g = \tanh(x_t U^g + s_{t-1} W^g) \quad \text{"candidate" hidden state:}$$

$$c_t = c_{t-1} \circ f + g \circ i$$

Cell internal memory

$$s_t = \tanh(c_t) \circ o$$

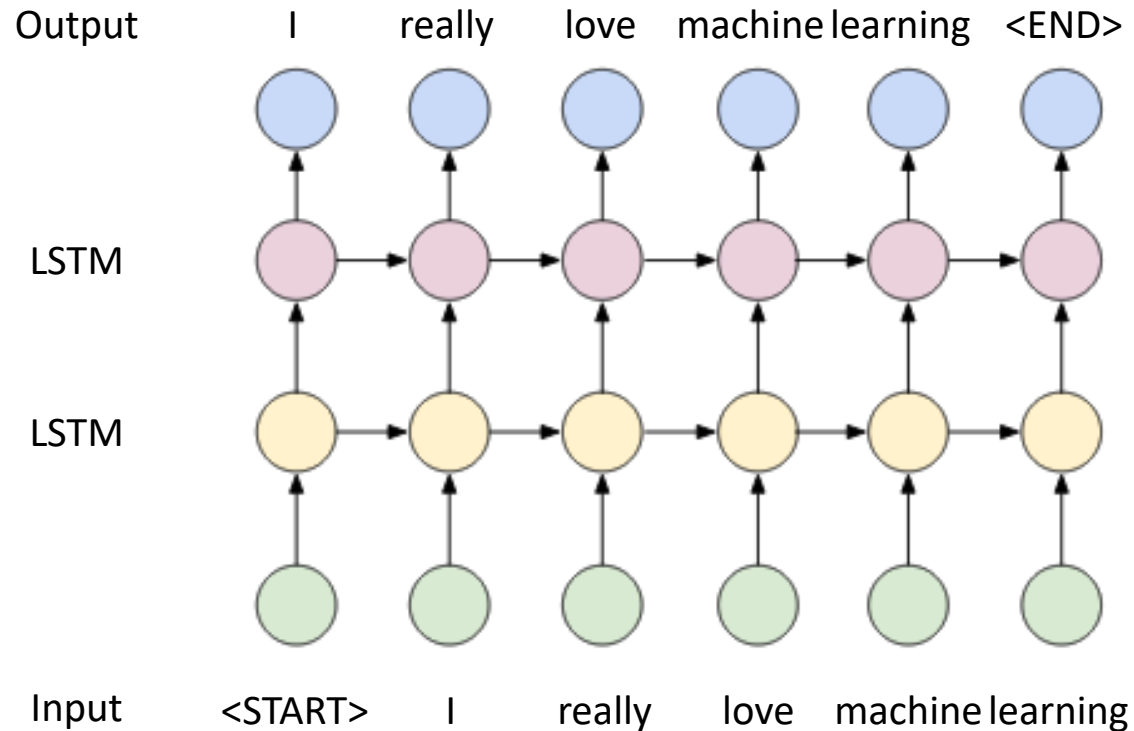
Hidden state

$$s_t = \tanh(x_t U + s_{t-1} W)$$

[<http://colah.github.io/posts/2015-08-Understanding-LSTMs/>]

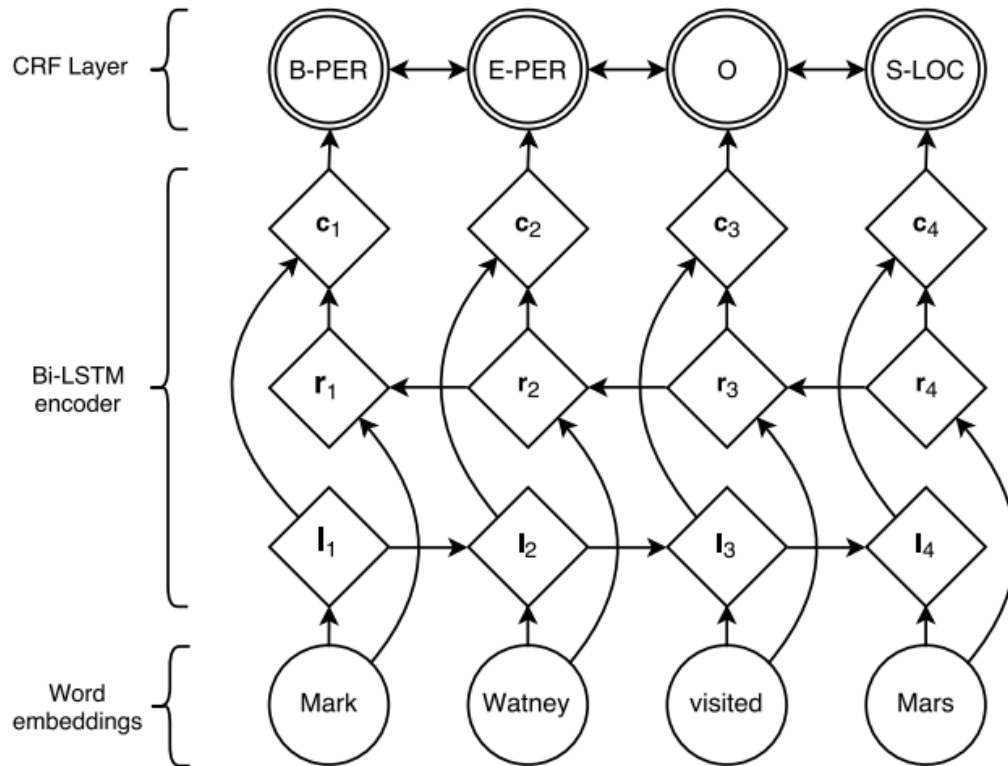
[Hochreiter, Sepp, and Jürgen Schmidhuber. "Long short-term memory." Neural computation 9.8 (1997): 1735-1780.]

Use Case: Text Generation



- A demo on character-level text generation
 - <http://cs.stanford.edu/people/karpathy/recurrentjs/>

Use Case: Named Entity Recognition



Word embedding

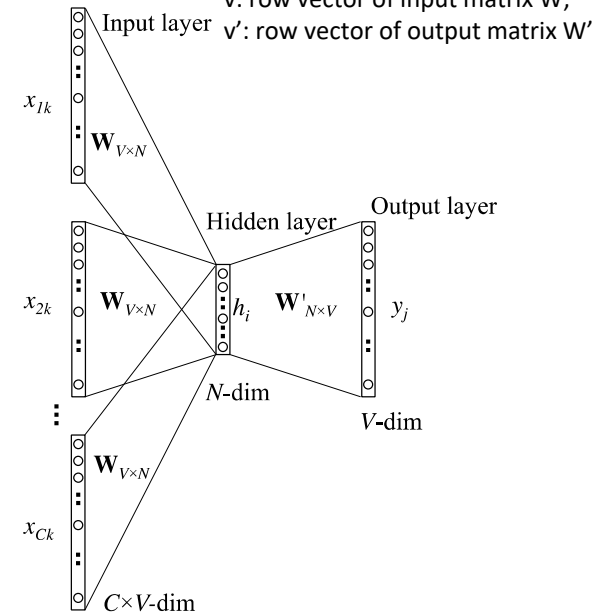
- From **bag of word** to **word embedding**
 - Use a real-valued vector in R^m to represent a word (concept)

$$v(\text{"cat"}) = (0.2, -0.4, 0.7, \dots)$$

$$v(\text{"mat"}) = (0.0, 0.6, -0.1, \dots)$$

- Continuous bag of word (CBOW) model (word2vec)
 - Input/output words x/y are **one-hot encoded**
 - Hidden layer is shared for all input words

V: vocabulary size;
 C: num. input words;
 v : row vector of input matrix W;
 v' : row vector of output matrix W'



N-dim Vector representation of a word

Hidden nodes:

$$\mathbf{h} = \frac{1}{C} \mathbf{W} \cdot (\mathbf{x}_1 + \mathbf{x}_2 + \dots + \mathbf{x}_C)$$

$$= \frac{1}{C} \cdot (\mathbf{v}_{w_1} + \mathbf{v}_{w_2} + \dots + \mathbf{v}_{w_C})$$

The cross-entropy loss:

$$E = -\log p(w_O | w_{I,1}, \dots, w_{I,C})$$

$$= -\mathbf{v}'_{w_O} \cdot \mathbf{h} + \log \sum_{j'=1}^V \exp(\mathbf{v}'_{w_{j'}} \cdot \mathbf{h})$$

The gradient updates:

$$\mathbf{v}'_{w_j}^{(\text{new})} = \mathbf{v}'_{w_j}^{(\text{old})} - \eta \cdot e_j \cdot \mathbf{h} \quad \text{for } j = 1, 2, \dots, V.$$

$$\mathbf{v}_{w_{I,c}}^{(\text{new})} = \mathbf{v}_{w_{I,c}}^{(\text{old})} - \frac{1}{C} \cdot \eta \cdot \mathbf{E} \mathbf{H}$$

Continuous bag of word (CBOW) model

$$\frac{\partial E}{\partial h_i} = \sum_{j=1}^V \frac{\partial E}{\partial u_j} \cdot \frac{\partial u_j}{\partial h_i} = \sum_{j=1}^V e_j \cdot w'_{ij} := \mathbf{E} \mathbf{H}_i$$

Rong, Xin. "word2vec parameter learning explained." arXiv preprint arXiv:1411.2738 (2014).

Mikolov, Tomas, et al. "Efficient estimation of word representations in vector space." arXiv preprint arXiv:1301.3781 (2013).

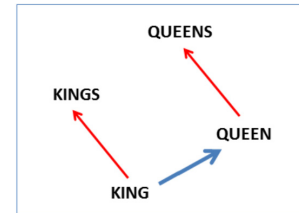
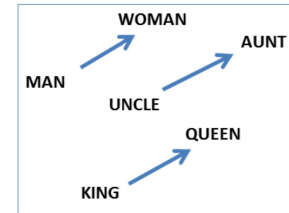
Remarkable properties from Word embedding

- Simple algebraic operations with the word vectors

Using $X = v(\text{"biggest"}) - v(\text{"big"}) + v(\text{"small"})$ as query and searching for the nearest word based on cosine distance results in $v(\text{"smallest"})$

$$v(\text{"woman"}) - v(\text{"man"}) \approx v(\text{"aunt"}) - v(\text{"uncle"})$$

$$v(\text{"woman"}) - v(\text{"man"}) \approx v(\text{"queen"}) - v(\text{"king"})$$

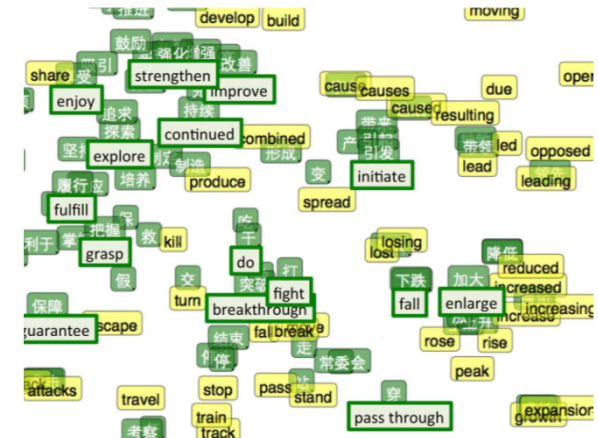


Word the relationship is defined by subtracting two word vectors, and the result is added to another word. Thus for example, **Paris - France + Italy = Rome.**

Vector offsets for gender relation

The singular/plural relation for two words

Relationship	Example 1	Example 2	Example 3
France - Paris	Italy: Rome	Japan: Tokyo	Florida: Tallahassee
big - bigger	small: larger	cold: colder	quick: quicker
Miami - Florida	Baltimore: Maryland	Dallas: Texas	Kona: Hawaii
Einstein - scientist	Messi: midfielder	Mozart: violinist	Picasso: painter
Sarkozy - France	Berlusconi: Italy	Merkel: Germany	Koizumi: Japan
copper - Cu	zinc: Zn	gold: Au	uranium: plutonium
Berlusconi - Silvio	Sarkozy: Nicolas	Putin: Medvedev	Obama: Barack
Microsoft - Windows	Google: Android	IBM: Linux	Apple: iPhone
Microsoft - Ballmer	Google: Yahoo	IBM: McNealy	Apple: Jobs
Japan - sushi	Germany: bratwurst	France: tapas	USA: pizza



Neural Language models

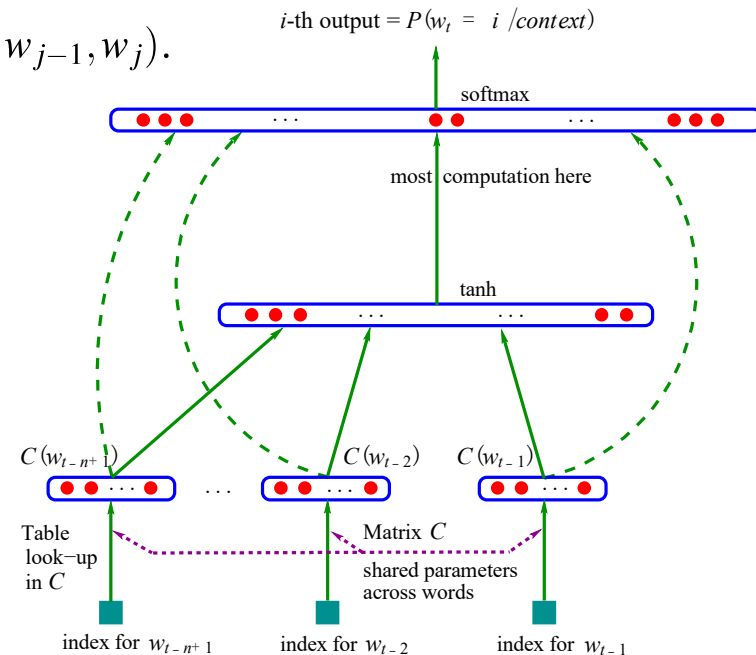
- *n-gram* model

- Construct conditional probabilities for the next word, given combinations of the last *n-1* words (*contexts*)

$$\hat{P}(w_t | w_1^{t-1}) \approx \hat{P}(w_t | w_{t-n+1}^{t-1}) \quad \text{where } w_i^j = (w_i, w_{i+1}, \dots, w_{j-1}, w_j).$$

- Neural language model

- associate with each word a *distributed word feature vector* for word embedding,
- express the *joint probability function* of word sequences using those vectors, and
- learn simultaneously the *word feature vectors* and the *parameters* of that *probability function*.



RNN based Language models

- The limitation of the feedforward network approach:
 - it has to fix the length context
- Recurrent network solves the issue
 - by **keeping a (hidden) context and updating over time**

x(t) is the input vector:

It is formed by **concatenating** vector $w(t)$ representing current word, and hidden state s at time $t - 1$. $w(t)$ is one hot encoder of a word

s(t) is state of the network (the hidden layer):

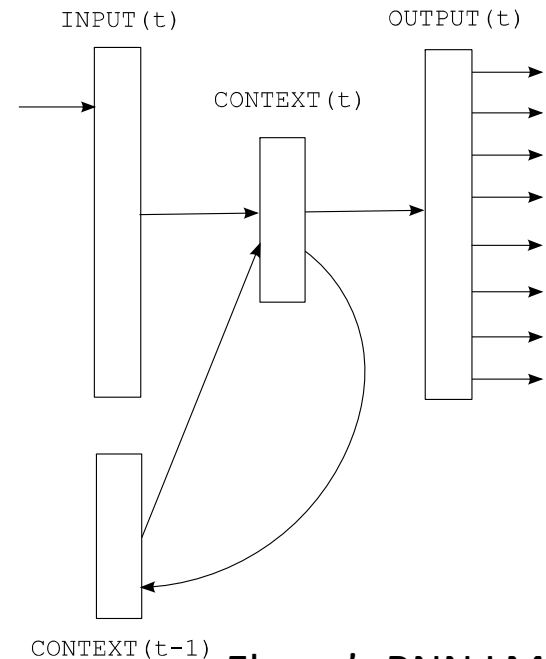
$$x(t) = [w(t), s(t-1)]$$

$$s_j(t) = f \left(\sum_i x_i(t) u_{ji} \right)$$

$$y_k(t) = g \left(\sum_j s_j(t) v_{kj} \right)$$

output is denoted as y(t):

Sigmoid for hidden layer $f(z) = \frac{1}{1 + e^{-z}}$ **Softmax for output layer** $g(z_m) = \frac{e^{z_m}}{\sum_k e^{z_k}}$



Elman's RNN LM

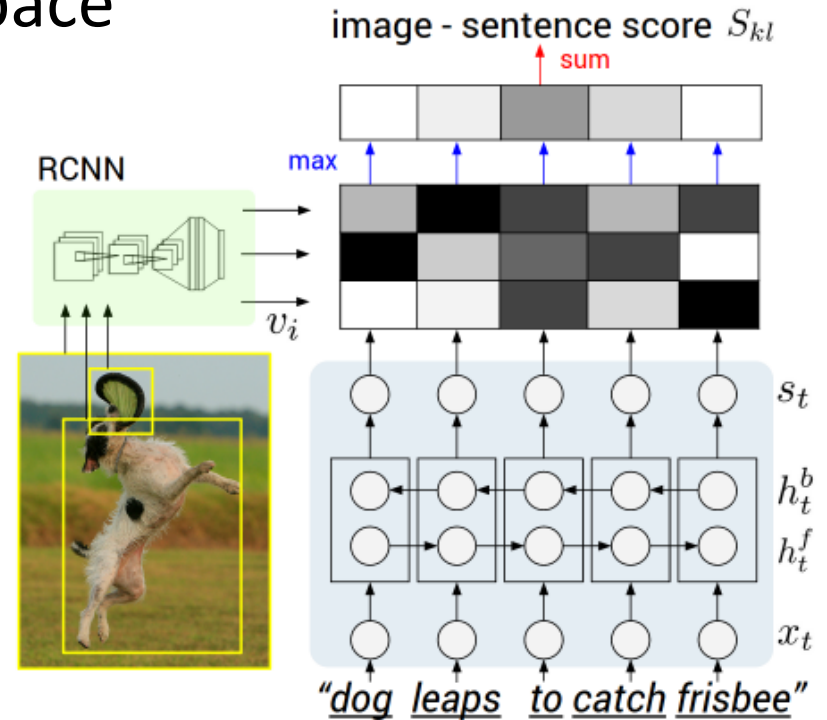
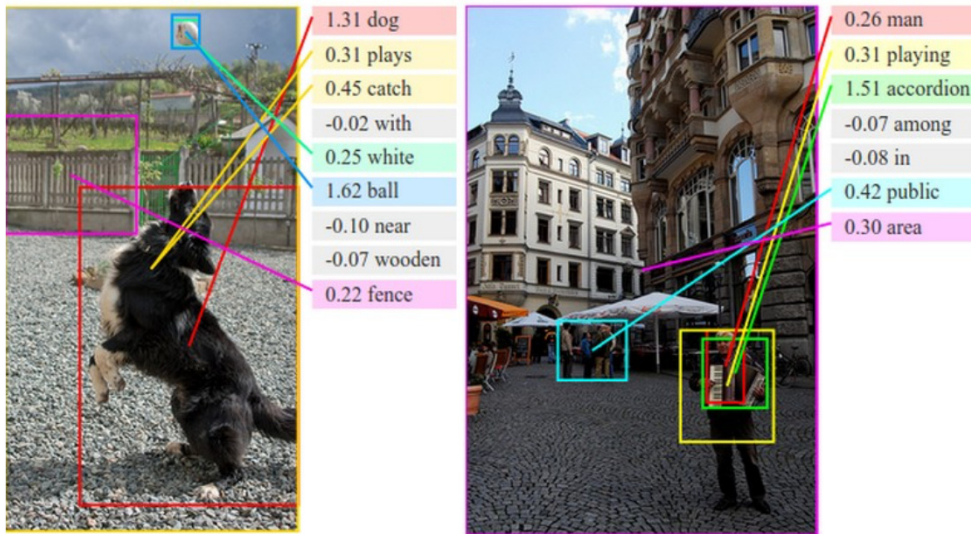
Mikolov, Tomas, et al. "Recurrent neural network based language model." INTERSPEECH. Vol. 2. 2010.

Elman J L. Finding structure in time[J]. Cognitive science, 1990, 14(2): 179-211.

Learning to align visual and language data

- Regional CNN + Bi-directional RNN

- associates the two modalities through a common, multimodal embedding space

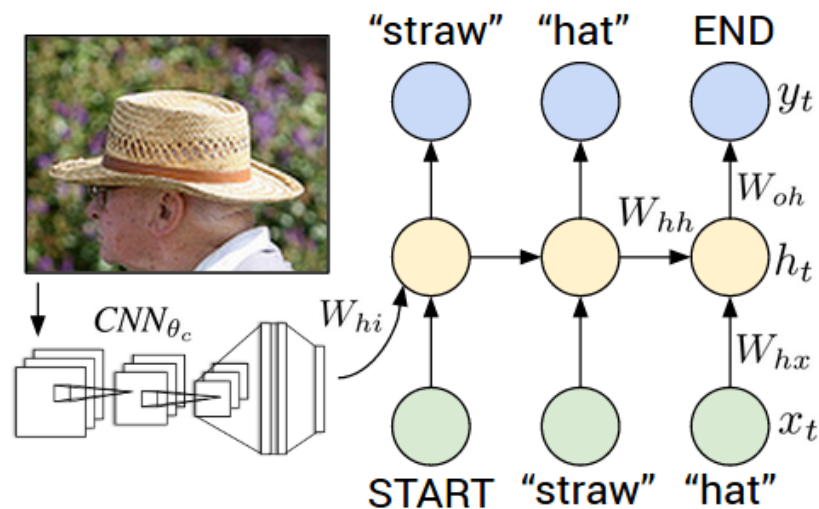


Learning to generate image descriptions

- **Trained CNN** on images + **RNN** with sentence
 - The RNN takes a word, the previous context and defines a distribution over the next word
 - The RNN is conditioned on the image information at the first time step
 - START and END are special tokens.



"two young girls are playing with lego toy."



Summary

- Universal Approximation: two-layer neural networks can approximate any functions
- Backpropagation is the most important training scheme for multi-layer neural networks so far
- Deep learning, i.e. deep architecture of NN trained with big data, works incredibly well
- Neural networks built with other machine learning models achieve further success

Reference Materials

- Prof. Geoffery Hinton's Coursera course
 - <https://www.coursera.org/learn/neural-networks>
- Prof. Jun Wang's DL tutorial in UCL ([special thanks](#))
 - <http://www.slideshare.net/JunWang5/deep-learning-61493694>
- Prof. Fei-fei Li's CS231n in Stanford
 - <http://cs231n.stanford.edu/>
- Prof. Kai Yu's DL Course in SJTU
 - <http://speechlab.sjtu.edu.cn/~kyu/node/10>
- Michael Nielsen's online DL book
 - <http://neuralnetworksanddeeplearning.com/>
- Research Blogs
 - Andrej Karpathy: <http://karpathy.github.io/>
 - Christopher Olah: <http://colah.github.io/>