2019 EE448, Big Data Mining, Lecture 9

Learning to Rank

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Content of This Course

- Another ML problem: ranking
- Learning to rank
- Pointwise methods
- Pairwise methods
- Listwise methods

Ranking Problem

Learning to rank

Pointwise methods

Pairwise methods

Listwise methods

Sincerely thank Dr. Tie-Yan Liu

The Probability Ranking Principle

- https://nlp.stanford.edu/IR-book/html/htmledition/the-probability-ranking-principle-1.html

Regression and Classification

- Supervised learning

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \mathcal{L}(y_i, f_\theta(x_i))$$

- Two major problems for supervised learning
  - Regression

$$\mathcal{L}(y_i, f_\theta(x_i)) = \frac{1}{2}(y_i - f_\theta(x_i))^2$$

  - Classification

$$\mathcal{L}(y_i, f_\theta(x_i)) = -y_i \log f_\theta(x_i) - (1 - y_i) \log(1 - f_\theta(x_i))$$

Learning to Rank Problem

- Input: a set of instances

$$X = \{x_1, x_2, \ldots, x_n\}$$

- Output: a rank list of these instances

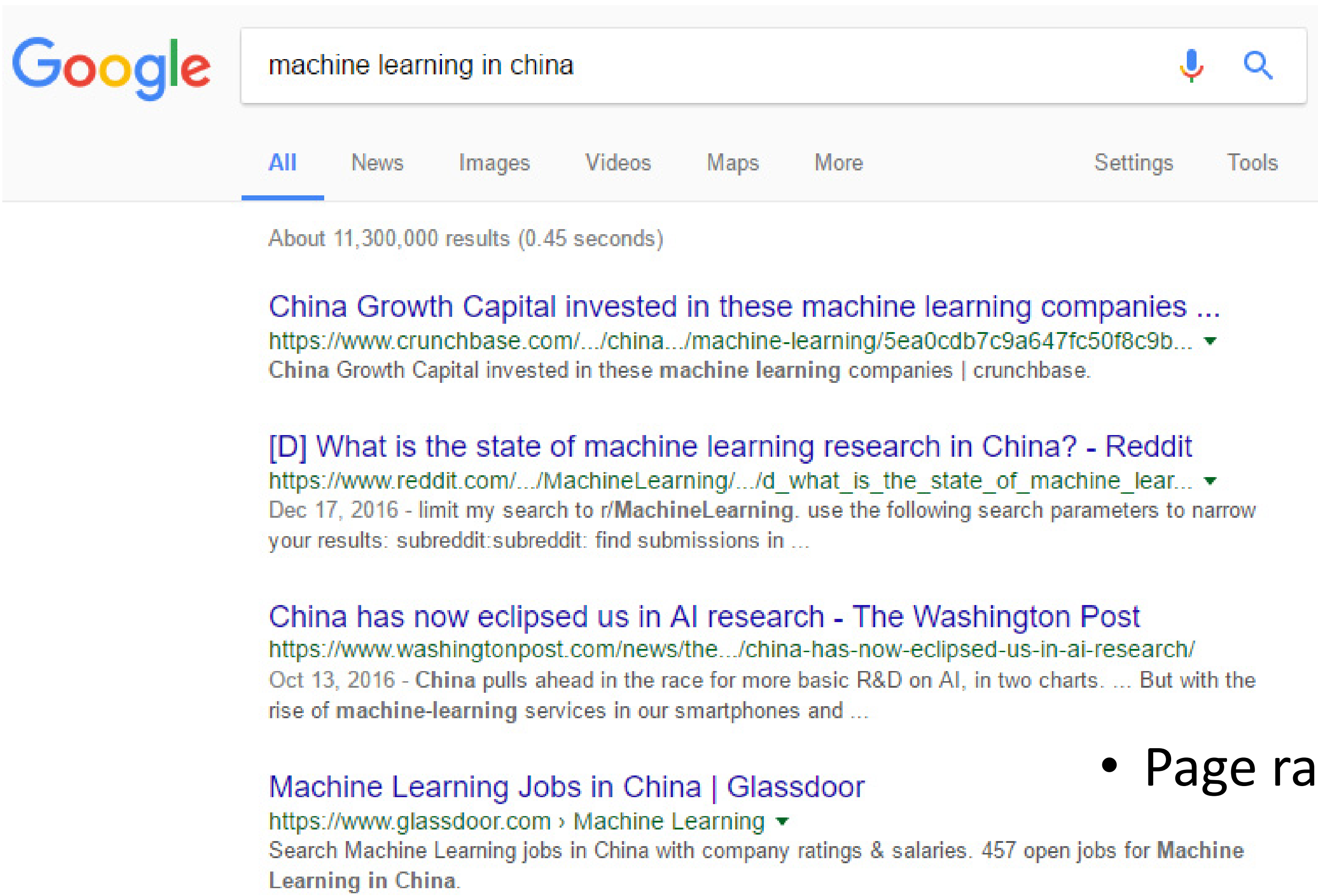
$$\hat{Y} = \{x_{r_1}, x_{r_2}, \ldots, x_{r_n}\}$$

- Ground truth: a correct ranking of these instances

$$Y = \{x_{y_1}, x_{y_2}, \ldots, x_{y_n}\}$$

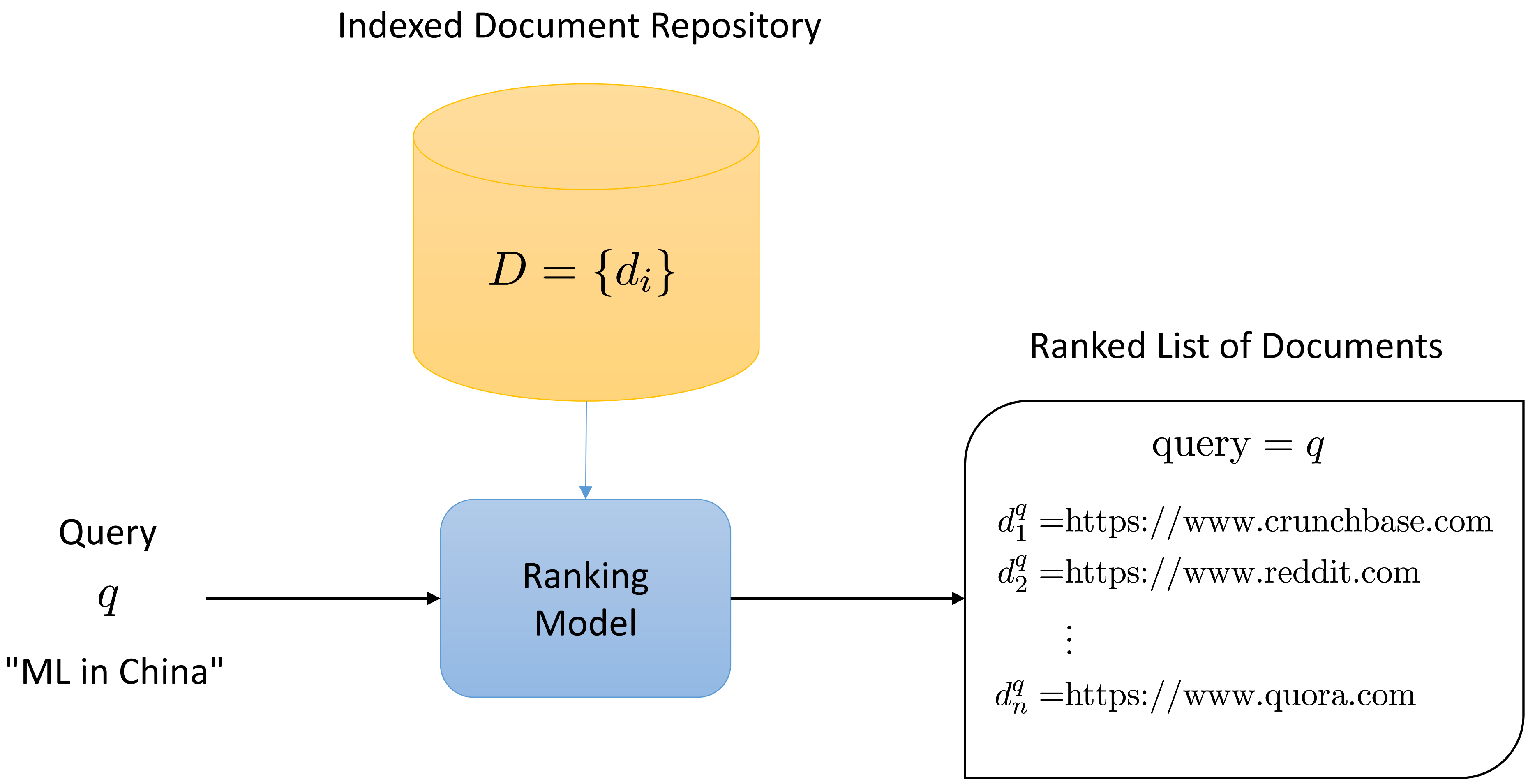
A Typical Application

- Webpage ranking given a query



- Page ranking

Webpage Ranking



Model Perspective

- In most existing work, learning to rank is defined as having the following two properties
  - Feature-based
    - Each instance (e.g. query-document pair) is represented with a list of features
  - Discriminative training
    - Estimate the relevance given a query-document pair
    - Rank the documents based on the estimation

$$y_i = f_\theta(x_i)$$

Learning to Rank

- Input: features of query and documents
  - Query, document, and combination features
- Output: the documents ranked by a scoring function

$$y_i = f_\theta(x_i)$$

- Objective: relevance of the ranking list
  - Evaluation metrics: NDCG, MAP, MRR…
- Training data: the query-doc features and relevance ratings

Training Data

- The query-doc features and relevance ratings

Query='ML in China'

Features

Rating	Document	Query Length	Doc PageRank	Doc Length	Title Rel.	Content Rel.
3	d_1=http://crunchbase.com	0.30	0.61	0.47	0.54	0.76
5	d_2=http://reddit.com	0.30	0.81	0.76	0.91	0.81
4	d_3=http://quora.com	0.30	0.86	0.56	0.96	0.69

Query features

Document features

Query-doc features

Learning to Rank Approaches

- Learn (not define) a scoring function to optimally rank the documents given a query

- Pointwise
  - Predict the absolute relevance (e.g. RMSE)
- Pairwise
  - Predict the ranking of a document pair (e.g. AUC)
- Listwise
  - Predict the ranking of a document list (e.g. Cross Entropy)

Tie-Yan Liu. Learning to Rank for Information Retrieval. Springer 2011.
http://www.cda.cn/uploadfile/image/20151220/20151220115436_46293.pdf

Pointwise Approaches

- Predict the expert ratings
  - As a regression problem

$$y_i = f_\theta(x_i)$$

$$\min_\theta \frac{1}{2N} \sum_{i=1}^{N} (y_i - f_\theta(x_i))^2$$

Query='ML in China'

Features

Rating	Document	Query Length	Doc PageRank	Doc Length	Title Rel.	Content Rel.
3	d_1=http://crunchbase.com	0.30	0.61	0.47	0.54	0.76
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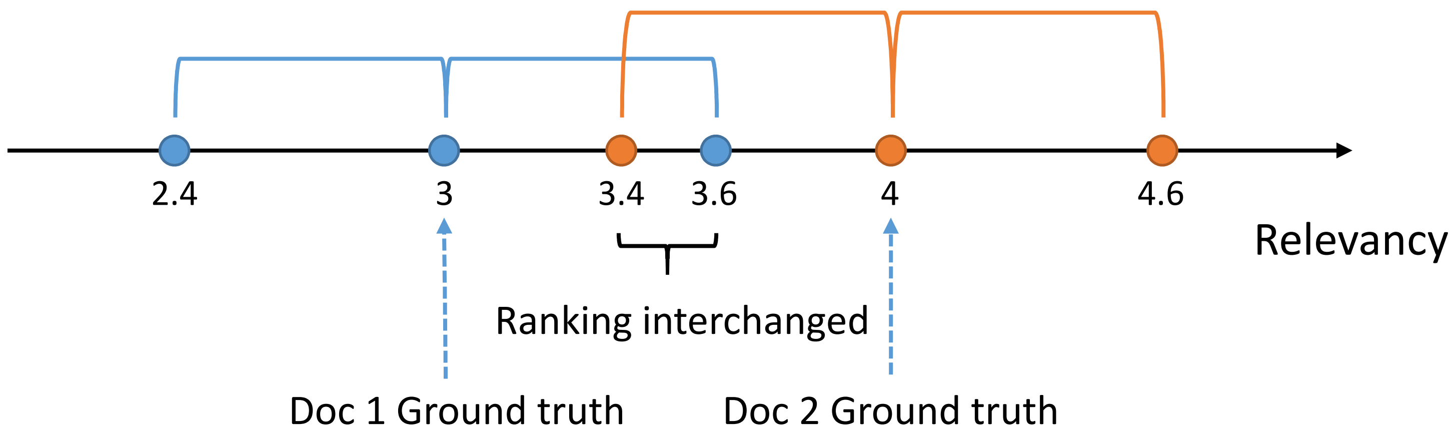
4	d_3=http://quora.com	0.30	0.86	0.56	0.96	0.69

Point Accuracy != Ranking Accuracy



- Same square error might lead to different rankings

Pairwise Approaches

- Not care about the absolute relevance but the relative preference on a document pair

- A binary classification

$$q^{(i)}$$

$$\begin{bmatrix} d_1^{(i)}, 5 \\ d_2^{(i)}, 3 \\ \vdots \\ d_{n^{(i)}}^{(i)}, 2 \end{bmatrix} \xrightarrow{\text{Transform}} \begin{array}{c} q^{(i)} \\ \left\{ \underset{5>3}{(d_1^{(i)}, d_2^{(i)})}, \underset{5>2}{(d_1^{(i)}, d_{n^{(i)}}^{(i)})}, \ldots, \underset{3>2}{(d_2^{(i)}, d_{n^{(i)}}^{(i)})} \right\} \end{array}$$

Binary Classification for Pairwise Ranking

- Given a query q and a pair of documents (d_i, d_j)
- Target probability $y_{i,j} = \begin{cases} 1 & \text{if } i \triangleright j \\ 0 & \text{otherwise} \end{cases}$
- Modeled probability

$$P_{i,j} = P(d_i \triangleright d_j | q) = \frac{\exp(o_{i,j})}{1 + \exp(o_{i,j})}$$

$$o_{i,j} \equiv f(x_i) - f(x_j)$$

x_i is the feature vector of (q, d_i)

- Cross entropy loss

$$\mathcal{L}(q, d_i, d_j) = -y_{i,j} \log P_{i,j} - (1 - y_{i,j}) \log(1 - P_{i,j})$$

RankNet

- The scoring function $f_\theta(x_i)$ is implemented by a neural network
- Modeled probability $P_{i,j} = P(d_i \triangleright d_j | q) = \dfrac{\exp(o_{i,j})}{1 + \exp(o_{i,j})}$

$$o_{i,j} \equiv f(x_i) - f(x_j)$$

- Cross entropy loss

$$\mathcal{L}(q, d_i, d_j) = -y_{i,j} \log P_{i,j} - (1 - y_{i,j}) \log(1 - P_{i,j})$$

- Gradient by chain rule

$$\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial \theta} = \frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial P_{i,j}} \frac{\partial P_{i,j}}{\partial o_{i,j}} \frac{\partial o_{i,j}}{\partial \theta}$$

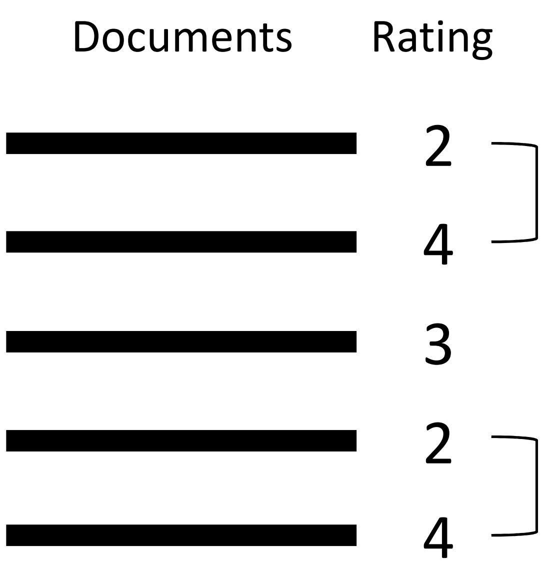
BP in NN

$$= \frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial P_{i,j}} \frac{\partial P_{i,j}}{\partial o_{i,j}} \Big(\frac{\partial f_\theta(x_i)}{\partial \theta} - \frac{\partial f_\theta(x_j)}{\partial \theta} \Big)$$

Burges, Christopher JC, Robert Ragno, and Quoc Viet Le. "Learning to rank with nonsmooth cost functions." *NIPS*. Vol. 6. 2006.

Shortcomings of Pairwise Approaches

- Each document pair is regarded with the same importance



Same pair-level error but different list-level error

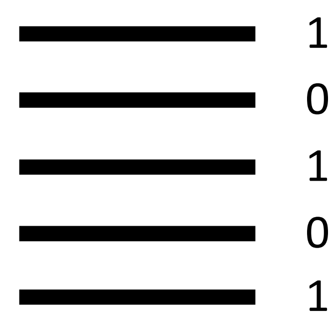
Ranking Evaluation Metrics

- For binary labels $y_i = \begin{cases} 1 & \text{if } d_i \text{ is relevant with } q \\ 0 & \text{otherwise} \end{cases}$

- Precision@*k* for query *q*

$$P@k = \frac{\#\{\text{relevant documents in top } k \text{ results}\}}{k}$$

- Average precision for query *q*



$$AP = \frac{\sum_k P@k \cdot y_{i(k)}}{\#\{\text{relevant documents}\}}$$

  - *i*(*k*) is the document id at *k*-th position

$$AP = \frac{1}{3} \cdot \left(\frac{1}{1} + \frac{2}{3} + \frac{3}{5}\right)$$

- Mean average precision (MAP): average over all queries

Ranking Evaluation Metrics

- For score labels, e.g.,

$$y_i \in \{0, 1, 2, 3, 4\}$$

- Normalized discounted cumulative gain (NDCG@*k*) for query *q*

$$NDCG@k = Z_k \sum_{j=1}^{k} \frac{2^{y_{i(j)}} - 1}{\log(j+1)}$$

Gain

Discount

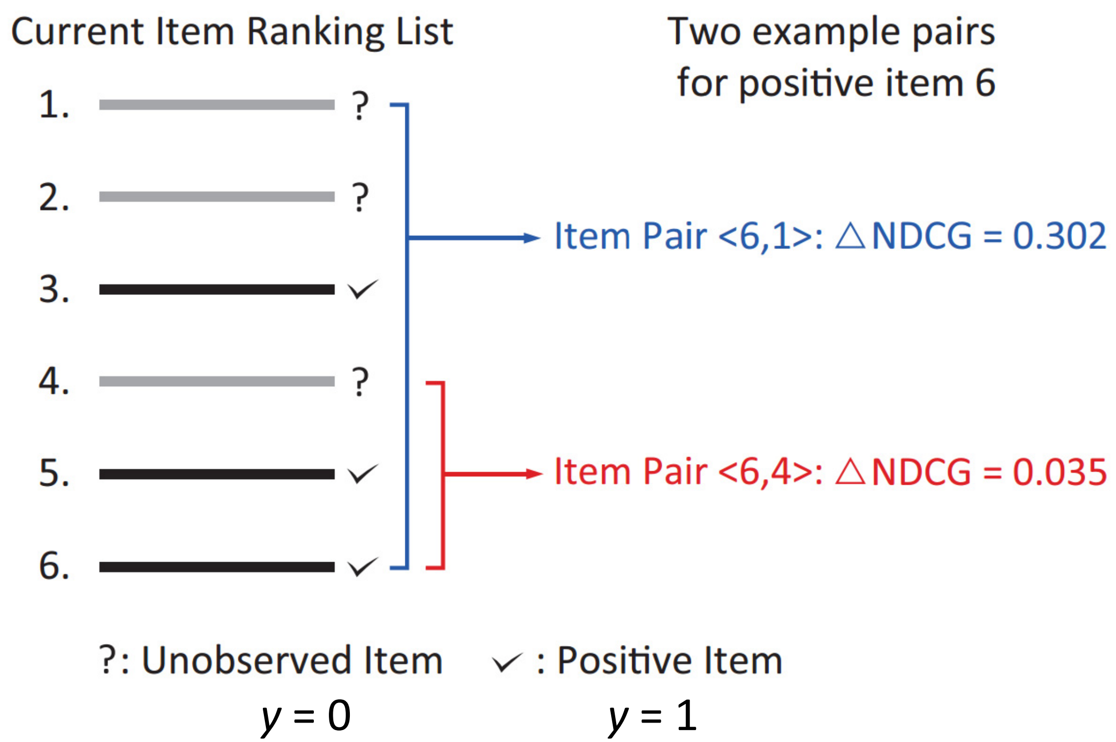
Normalizer

  - $i(j)$ is the document id at j-th position
  - Z_k is set to normalize the DCG of the ground truth ranking as 1

Shortcomings of Pairwise Approaches

- Same pair-level error but different list-level error

$$NDCG@k = Z_k \sum_{j=1}^{k} \frac{2^{y_{i(j)}} - 1}{\log(j+1)}$$



Listwise Approaches

- Training loss is directly built based on the difference between the prediction list and the ground truth list
- Straightforward target
  - Directly optimize the ranking evaluation measures
- Complex model

ListNet

- Train the score function $y_i = f_\theta(x_i)$
- Rankings generated based on $\{y_i\}_{i=1\ldots n}$
- Each possible *k*-length ranking list has a probability

$$P_f([j_1, j_2, \ldots, j_k]) = \prod_{t=1}^{k} \frac{\exp(f(x_{j_t}))}{\sum_{l=t}^{n} \exp(f(x_{j_l}))}$$

- List-level loss: cross entropy between the predicted distribution and the ground truth

$$\mathcal{L}(\boldsymbol{y}, f(\boldsymbol{x})) = -\sum_{g \in \mathcal{G}_k} P_y(g) \log P_f(g)$$

- Complexity: many possible rankings

Cao, Zhe, et al. "Learning to rank: from pairwise approach to listwise approach." *Proceedings of the 24th international conference on Machine learning*. ACM, 2007.

Distance between Ranked Lists

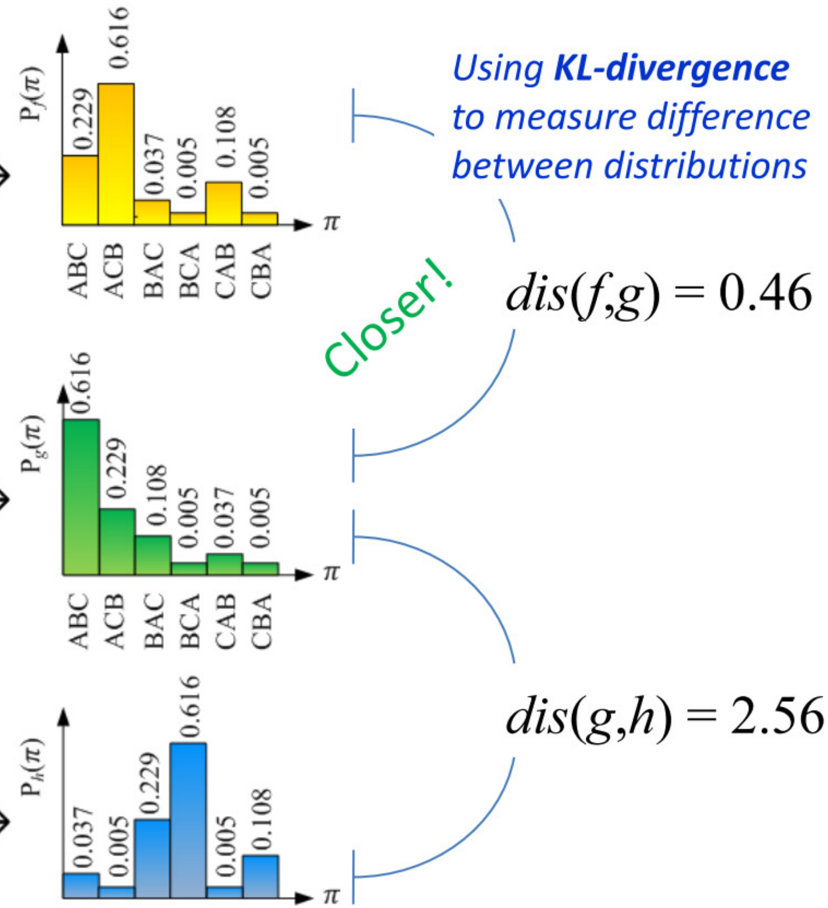
- A similar distance: KL divergence

$\varphi = exp$

f: $f(A) = 3, f(B)=0, f(C)=1$;
Ranking by f: ABC $\leftrightarrow$

g: $g(A) = 6, g(B)=4, g(C)=3$;
Ranking by g: ABC $\leftrightarrow$

h: $h(A) = 4, h(B)=6, h(C)=3$;
Ranking by h: ACB $\leftrightarrow$



Pairwise vs. Listwise

- Pairwise approach shortcoming
  - Pair-level loss is away from IR list-level evaluations
- Listwise approach shortcoming
  - Hard to define a list-level loss under a low model complexity
- A good solution: LambdaRank
  - Pairwise training with listwise information

Burges, Christopher JC, Robert Ragno, and Quoc Viet Le. "Learning to rank with nonsmooth cost functions." *NIPS*. Vol. 6. 2006.

LambdaRank

- Pairwise approach gradient

$$o_{i,j} \equiv f(x_i) - f(x_j)$$

$$\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial \theta} = \underbrace{\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial P_{i,j}} \frac{\partial P_{i,j}}{\partial o_{i,j}}}_{\lambda_{i,j}} \Big(\frac{\partial f_\theta(x_i)}{\partial \theta} - \frac{\partial f_\theta(x_i)}{\partial \theta} \Big)$$

Pairwise ranking loss

Scoring function itself

Current ranking list

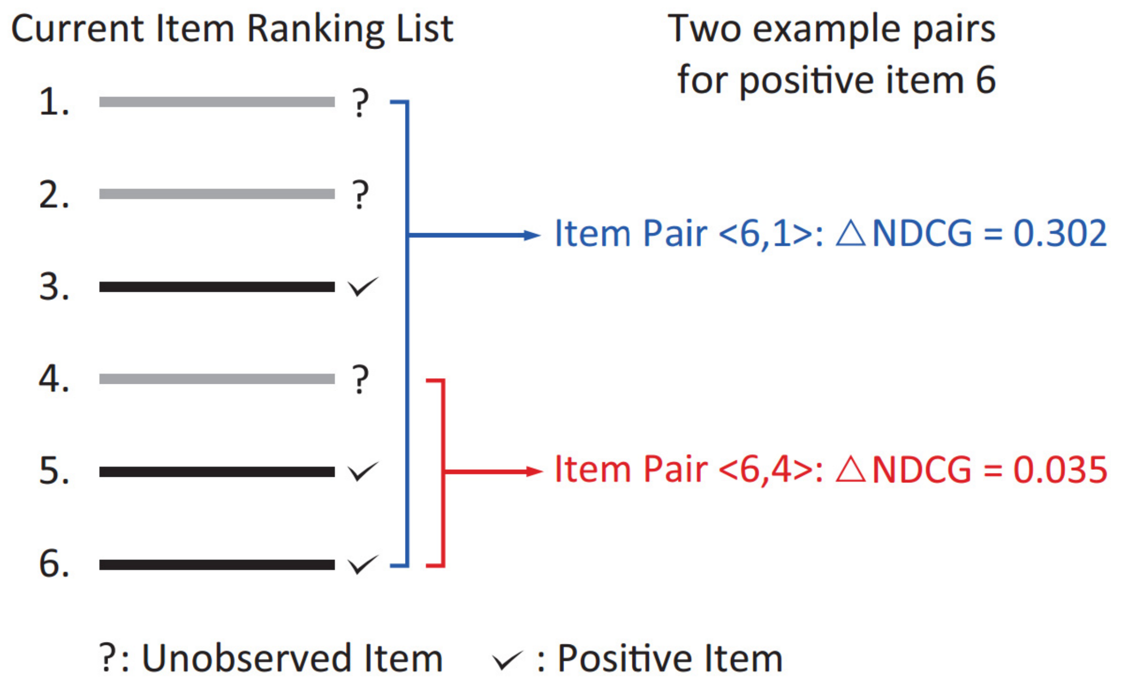
- LambdaRank basic idea
    - Add listwise information into $\lambda_{i,j}$ as $h(\lambda_{i,j}, g_q)$

$$\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial \theta} = h(\lambda_{i,j}, g_q) \Big(\frac{\partial f_\theta(x_i)}{\partial \theta} - \frac{\partial f_\theta(x_j)}{\partial \theta} \Big)$$

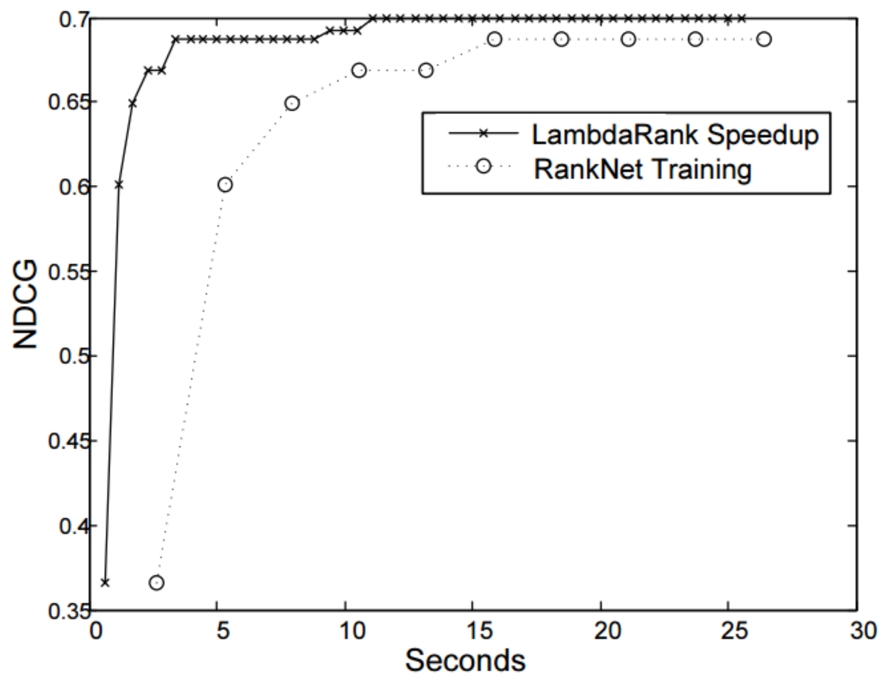
LambdaRank for Optimizing NDCG

- A choice of Lambda for optimize NDCG

$$h(\lambda_{i,j}, g_q) = \lambda_{i,j} \Delta NDCG_{i,j}$$



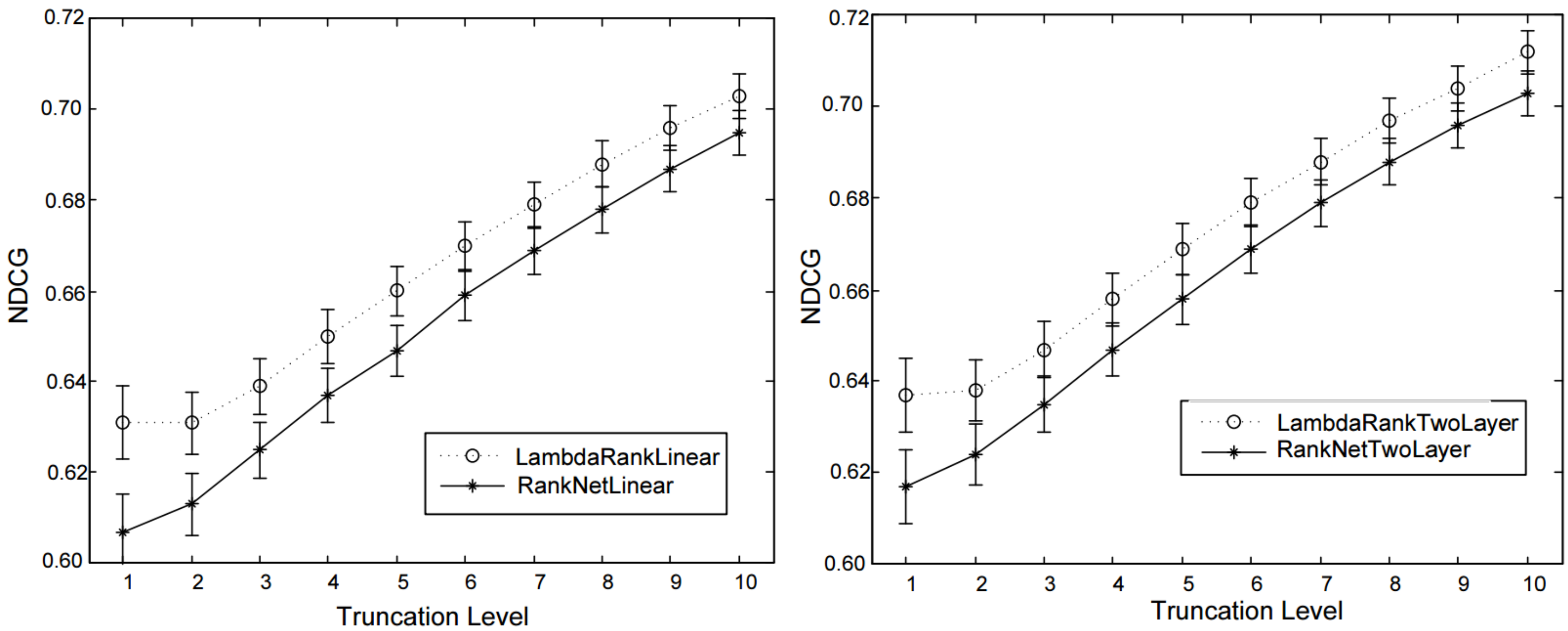
LambdaRank vs. RankNet



Linear nets

Burges, Christopher JC, Robert Ragno, and Quoc Viet Le. "Learning to rank with nonsmooth cost functions." *NIPS*. Vol. 6. 2006.

LambdaRank vs. RankNet



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Summary of Learning to Rank

- Pointwise, pairwise and listwise approaches for learning to rank
- Pairwise approaches are still the most popular
  - A balance of ranking effectiveness and training efficiency
- LambdaRank is a pairwise approach with list-level information
  - Easy to implement, easy to improve and adjust