2019 CS420, Machine Learning, Lecture 7

Ranking and Filtering

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http://wnzhang.net/teaching/cs420/index.html

Content of This Course

- Another ML problem: ranking
    - Learning to rank
    - Pointwise methods
    - Pairwise methods
    - Listwise methods

- A data mining application: Recommendation
    - Overview
    - Collaborative filtering
    - Rating prediction
    - Top-N ranking

Ranking Problem

Learning to rank
Pointwise methods
Pairwise methods
Listwise methods

Sincerely thank Dr. Tie-Yan Liu

Regression and Classification

- Supervised learning

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \mathcal{L}(y_i, f_\theta(x_i))$$

- Two major problems for supervised learning
  - Regression

$$\mathcal{L}(y_i, f_\theta(x_i)) = \frac{1}{2}(y_i - f_\theta(x_i))^2$$

  - Classification

$$\mathcal{L}(y_i, f_\theta(x_i)) = -y_i \log f_\theta(x_i) - (1 - y_i) \log(1 - f_\theta(x_i))$$

Learning to Rank Problem

- Input: a set of instances

$$X = \{x_1, x_2, \ldots, x_n\}$$

- Output: a rank list of these instances

$$\hat{Y} = \{x_{r_1}, x_{r_2}, \ldots, x_{r_n}\}$$

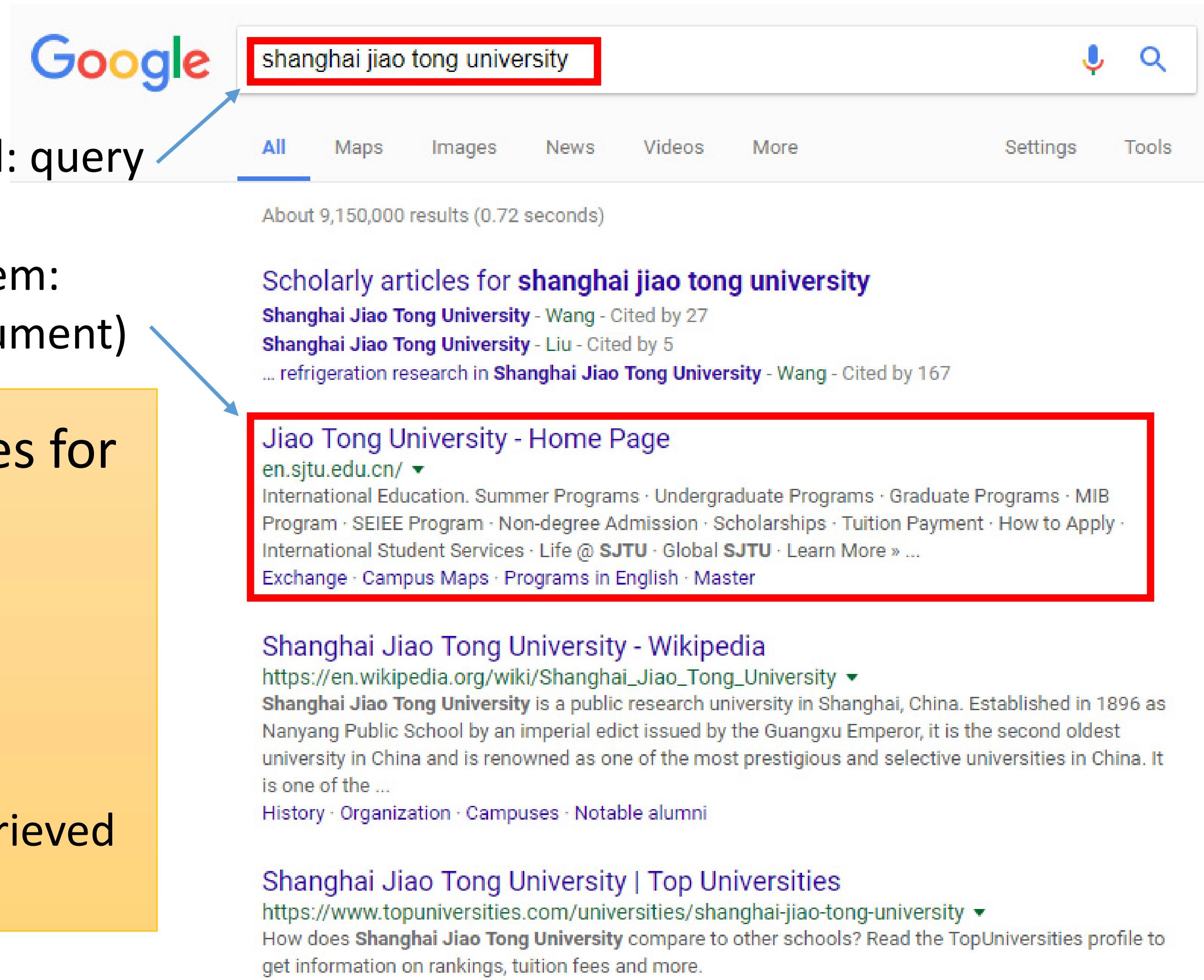
- Ground truth: a correct ranking of these instances

$$Y = \{x_{y_1}, x_{y_2}, \ldots, x_{y_n}\}$$

A Typical Application: Web Search Engines

Information need: query

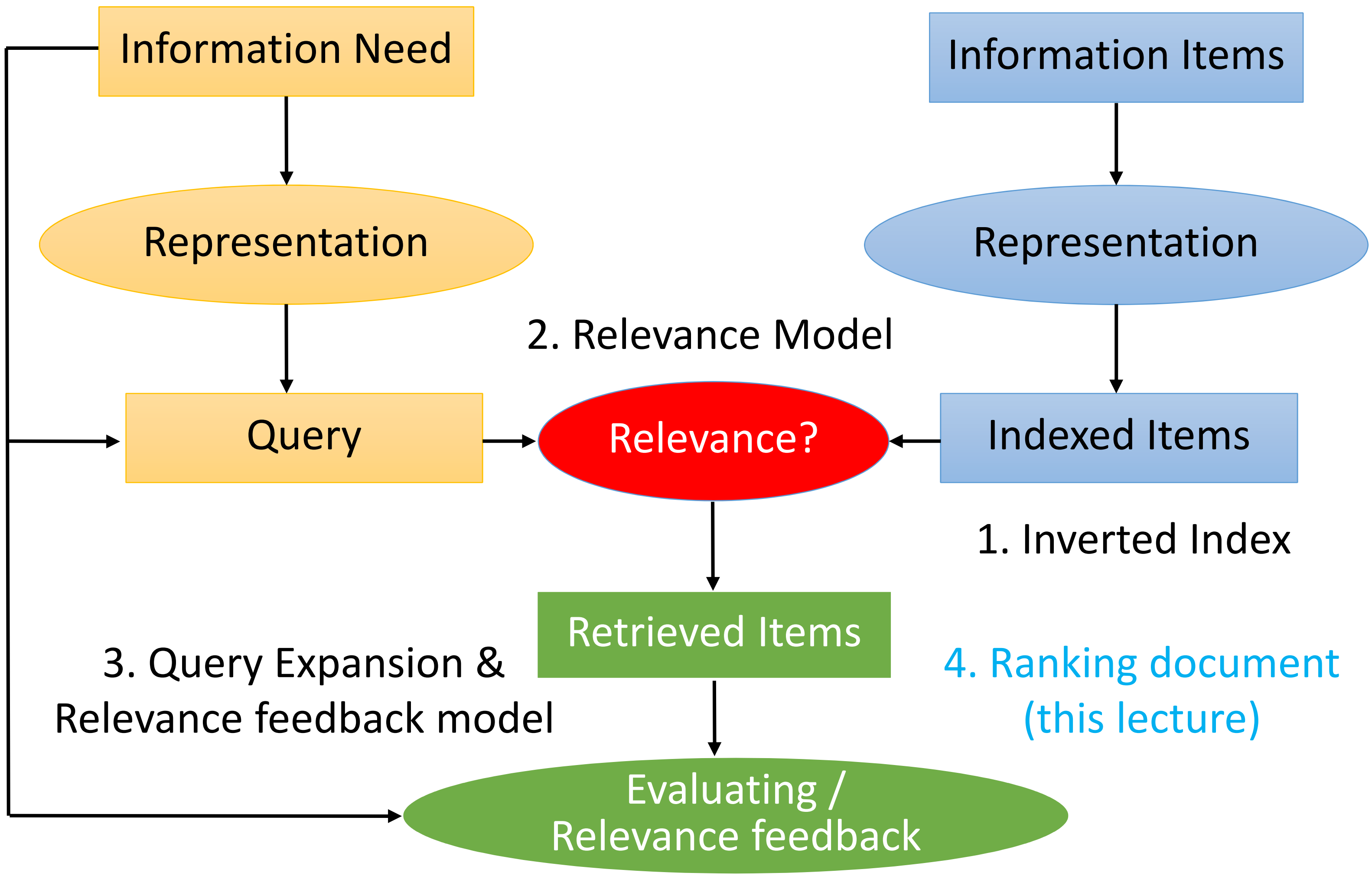
Information item: Webpage (or document)



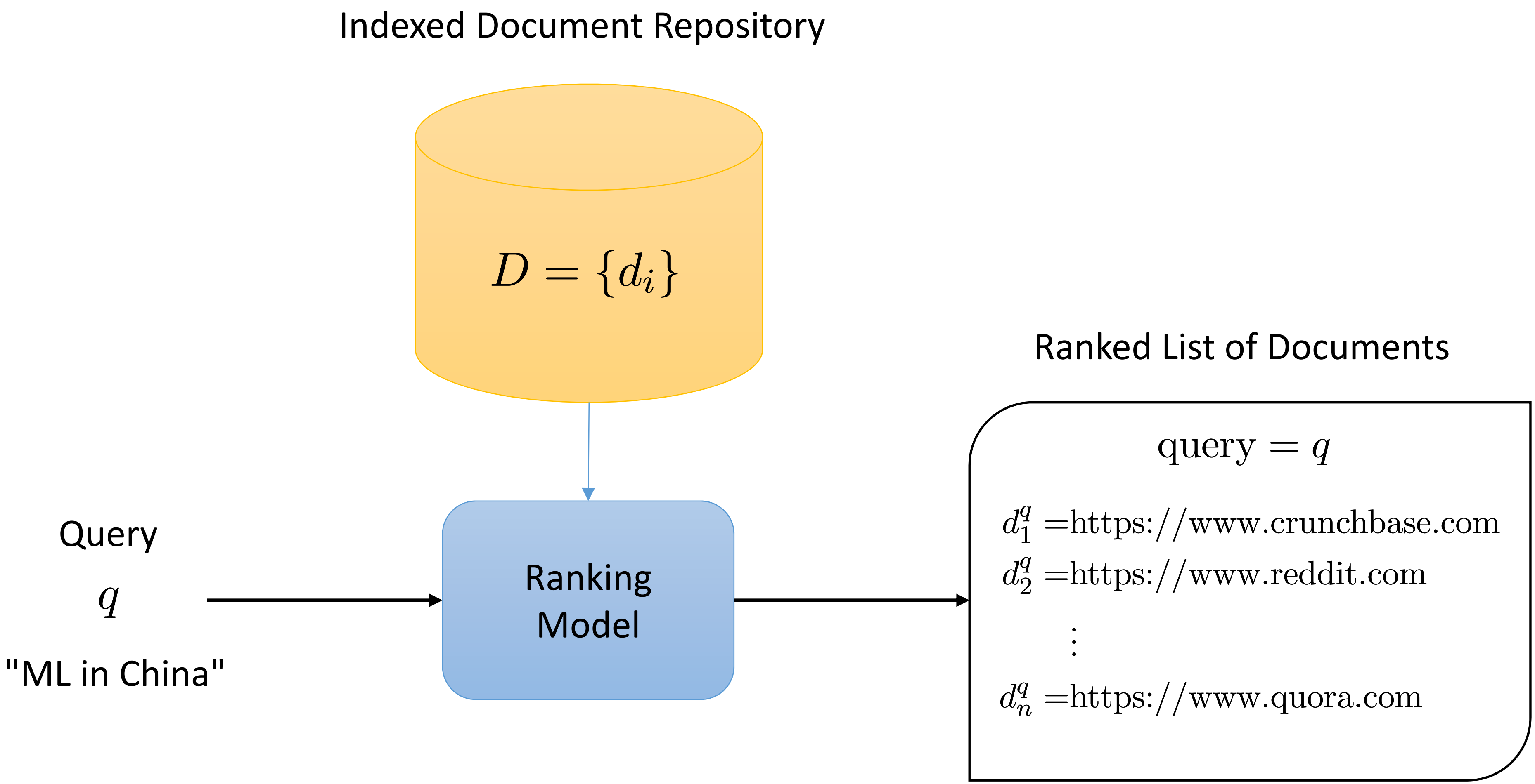
Two key stages for information retrieval:

- Retrieve the candidate documents
- Rank the retrieved documents

Overview Diagram of Information Retrieval



Webpage Ranking



Model Perspective

- In most existing work, learning to rank is defined as having the following two properties
  - Feature-based
    - Each instance (e.g. query-document pair) is represented with a list of features
  - Discriminative training
    - Estimate the relevance given a query-document pair
    - Rank the documents based on the estimation

$$y_i = f_\theta(x_i)$$

Learning to Rank

- Input: features of query and documents
  - Query, document, and combination features
- Output: the documents ranked by a scoring function

$$y_i = f_\theta(x_i)$$

- Objective: relevance of the ranking list
  - Evaluation metrics: NDCG, MAP, MRR…
- Training data: the query-doc features and relevance ratings

Training Data

- The query-doc features and relevance ratings

Query='ML in China'

Features

Rating	Document	Query Length	Doc PageRank	Doc Length	Title Rel.	Content Rel.
3	d_1=http://crunchbase.com	0.30	0.61	0.47	0.54	0.76
5	d_2=http://reddit.com	0.30	0.81	0.76	0.91	0.81
4	d_3=http://quora.com	0.30	0.86	0.56	0.96	0.69

Query features

Document features

Query-doc features

Learning to Rank Approaches

- Learn (not define) a scoring function to optimally rank the documents given a query

- Pointwise
  - Predict the absolute relevance (e.g. RMSE)
- Pairwise
  - Predict the ranking of a document pair (e.g. AUC)
- Listwise
  - Predict the ranking of a document list (e.g. Cross Entropy)

Tie-Yan Liu. Learning to Rank for Information Retrieval. Springer 2011.
http://www.cda.cn/uploadfile/image/20151220/20151220115436_46293.pdf

Pointwise Approaches

- Predict the expert ratings
  - As a regression problem

$$y_i = f_\theta(x_i)$$

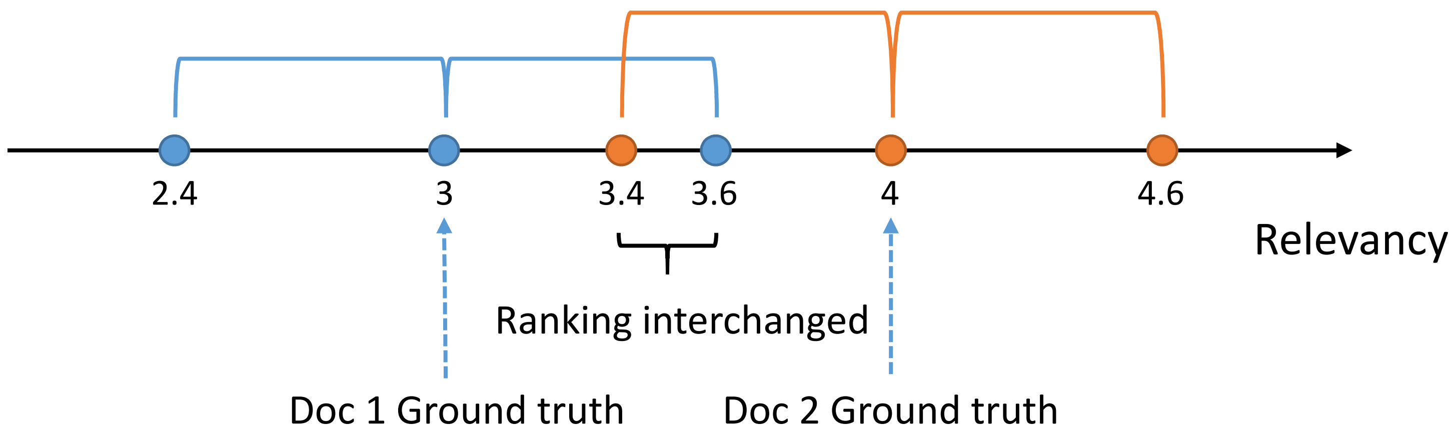
$$\min_\theta \frac{1}{2N} \sum_{i=1}^{N} (y_i - f_\theta(x_i))^2$$

Query='ML in China'

Features

Rating	Document	Query Length	Doc PageRank	Doc Length	Title Rel.	Content Rel.
3	d_1=http://crunchbase.com	0.30	0.61	0.47	0.54	0.76
5	d_2=http://reddit.com	0.30	0.81	0.76	0.91	0.81
4	d_3=http://quora.com	0.30	0.86	0.56	0.96	0.69

Point Accuracy != Ranking Accuracy



- Same square error might lead to different rankings

Pairwise Approaches

- Not care about the absolute relevance but the relative preference on a document pair

- A binary classification

$$q^{(i)}$$

$$\begin{bmatrix} d_1^{(i)}, 5 \\ d_2^{(i)}, 3 \\ \vdots \\ d_{n^{(i)}}^{(i)}, 2 \end{bmatrix} \xrightarrow{\text{Transform}} \begin{matrix} q^{(i)} \\ \left\{ (d_1^{(i)}, d_2^{(i)}), (d_1^{(i)}, d_{n^{(i)}}^{(i)}), \ldots, (d_2^{(i)}, d_{n^{(i)}}^{(i)}) \right\} \\ 5 > 3 \qquad\quad 5 > 2 \qquad\qquad\qquad 3 > 2 \end{matrix}$$

Binary Classification for Pairwise Ranking

- Given a query q and a pair of documents (d_i, d_j)
- Target probability $y_{i,j} = \begin{cases} 1 & \text{if } i \rhd j \\ 0 & \text{otherwise} \end{cases}$
- Modeled probability

$$P_{i,j} = P(d_i \rhd d_j | q) = \frac{\exp(o_{i,j})}{1 + \exp(o_{i,j})}$$

$$o_{i,j} \equiv f(x_i) - f(x_j)$$

x_i is the feature vector of (q, d_i)

- Cross entropy loss

$$\mathcal{L}(q, d_i, d_j) = -y_{i,j} \log P_{i,j} - (1 - y_{i,j}) \log(1 - P_{i,j})$$

RankNet

- The scoring function $f_\theta(x_i)$ is implemented by a neural network
- Modeled probability $P_{i,j} = P(d_i \rhd d_j | q) = \dfrac{\exp(o_{i,j})}{1 + \exp(o_{i,j})}$

  $o_{i,j} \equiv f(x_i) - f(x_j)$
- Cross entropy loss

$$\mathcal{L}(q, d_i, d_j) = -y_{i,j} \log P_{i,j} - (1 - y_{i,j}) \log(1 - P_{i,j})$$

- Gradient by chain rule

BP in NN

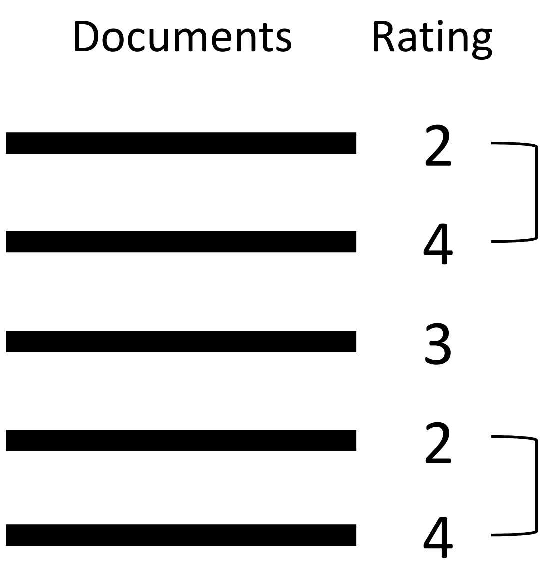
$$\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial \theta} = \frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial P_{i,j}} \frac{\partial P_{i,j}}{\partial o_{i,j}} \frac{\partial o_{i,j}}{\partial \theta}$$

$$= \frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial P_{i,j}} \frac{\partial P_{i,j}}{\partial o_{i,j}} \Big(\frac{\partial f_\theta(x_i)}{\partial \theta} - \frac{\partial f_\theta(x_j)}{\partial \theta} \Big)$$

Burges, Christopher JC, Robert Ragno, and Quoc Viet Le. "Learning to rank with nonsmooth cost functions." *NIPS*. Vol. 6. 2006.

Shortcomings of Pairwise Approaches

- Each document pair is regarded with the same importance



Same pair-level error but different list-level error

Ranking Evaluation Metrics

- For binary labels $y_i = \begin{cases} 1 & \text{if } d_i \text{ is relevant with } q \\ 0 & \text{otherwise} \end{cases}$

- Precision@*k* for query *q*

$$P@k = \frac{\#\{\text{relevant documents in top } k \text{ results}\}}{k}$$

- Average precision for query *q*

$$AP = \frac{\sum_k P@k \cdot y_{i(k)}}{\#\{\text{relevant documents}\}}$$

1
0
1
0
1

  - *i*(*k*) is the document id at *k*-th position

$$AP = \frac{1}{3} \cdot \left(\frac{1}{1} + \frac{2}{3} + \frac{3}{5}\right)$$

- Mean average precision (MAP): average over all queries

Ranking Evaluation Metrics

- For score labels, e.g.,

$$y_i \in \{0, 1, 2, 3, 4\}$$

- Normalized discounted cumulative gain (NDCG@*k*) for query *q*

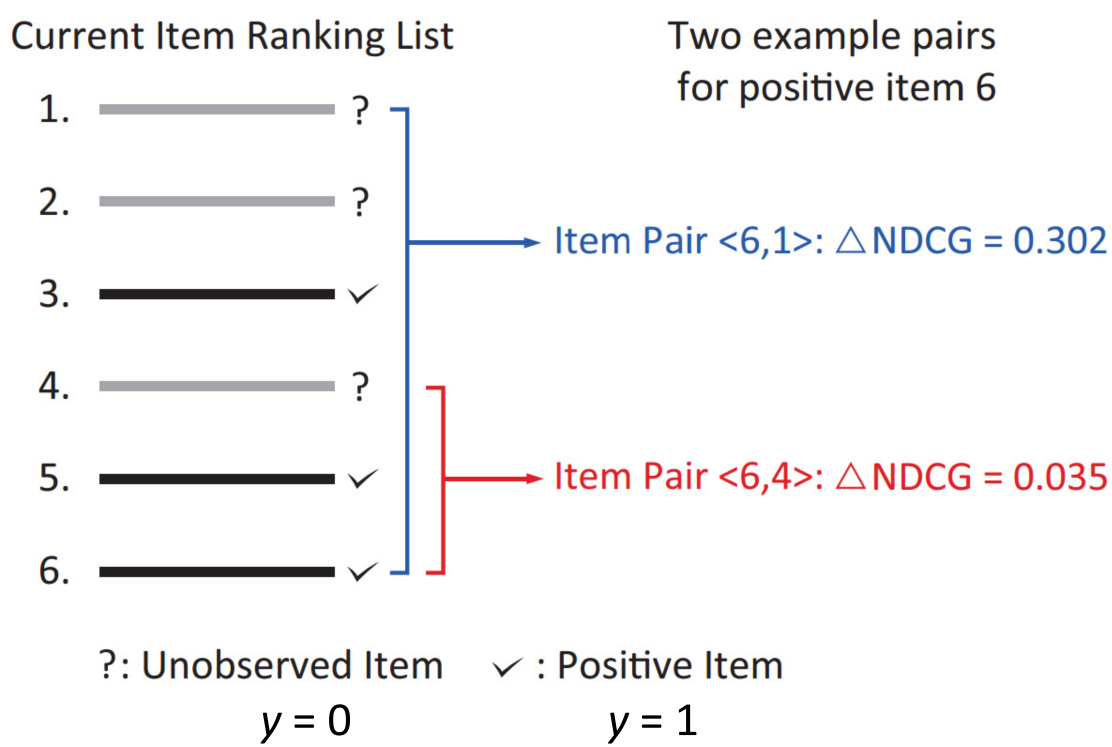
$$NDCG@k = \underbrace{Z_k}_{\text{Normalizer}} \sum_{j=1}^{k} \frac{2^{y_{i(j)}} - 1}{\log(j+1)} \quad \begin{matrix} \leftarrow \text{Gain} \\ \leftarrow \text{Discount} \end{matrix}$$

  - *i*(*j*) is the document id at *j*-th position
  - Z_k is set to normalize the DCG of the ground truth ranking as 1

Shortcomings of Pairwise Approaches

- Same pair-level error but different list-level error

$$NDCG@k = Z_k \sum_{j=1}^{k} \frac{2^{y_{i(j)}} - 1}{\log(j+1)}$$



Listwise Approaches

- Training loss is directly built based on the difference between the prediction list and the ground truth list

- Straightforward target
  - Directly optimize the ranking evaluation measures

- Complex model

ListNet

- Train the score function $y_i = f_\theta(x_i)$
- Rankings generated based on $\{y_i\}_{i=1\ldots n}$
- Each possible *k*-length ranking list has a probability

$$P_f([j_1, j_2, \ldots, j_k]) = \prod_{t=1}^{k} \frac{\exp(f(x_{j_t}))}{\sum_{l=t}^{n} \exp(f(x_{j_l}))}$$

- List-level loss: cross entropy between the predicted distribution and the ground truth

$$\mathcal{L}(\boldsymbol{y}, f(\boldsymbol{x})) = -\sum_{g \in \mathcal{G}_k} P_y(g) \log P_f(g)$$

- Complexity: many possible rankings

Cao, Zhe, et al. "Learning to rank: from pairwise approach to listwise approach." *Proceedings of the 24th international conference on Machine learning*. ACM, 2007.

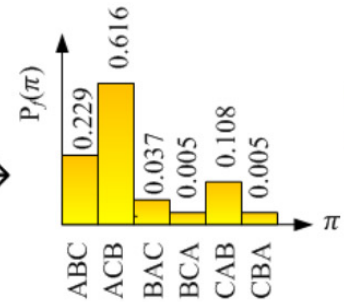
Distance between Ranked Lists

- A similar distance: KL divergence

$\varphi = exp$

f: $f(\text{A}) = 3, f(\text{B})=0, f(\text{C})=1$;
Ranking by f: ABC

$\leftrightarrow$



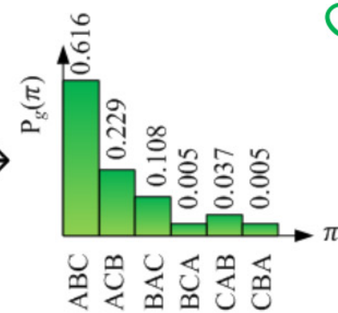
*Using **KL-divergence** to measure difference between distributions*

Closer!

$dis(f,g) = 0.46$

g: $g(\text{A}) = 6, g(\text{B})=4, g(\text{C})=3$;
Ranking by g: ABC

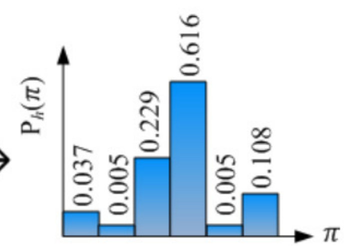
$\leftrightarrow$



$dis(g,h) = 2.56$

h: $h(\text{A}) = 4, h(\text{B})=6, h(\text{C})=3$;
Ranking by h: ACB

$\leftrightarrow$



Pairwise vs. Listwise

- Pairwise approach shortcoming
  - Pair-level loss is away from IR list-level evaluations
- Listwise approach shortcoming
  - Hard to define a list-level loss under a low model complexity

- A good solution: LambdaRank
  - Pairwise training with listwise information

Burges, Christopher JC, Robert Ragno, and Quoc Viet Le. "Learning to rank with nonsmooth cost functions." *NIPS*. Vol. 6. 2006.

LambdaRank

- Pairwise approach gradient

$$o_{i,j} \equiv f(x_i) - f(x_j)$$

$$\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial \theta} = \underbrace{\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial P_{i,j}} \frac{\partial P_{i,j}}{\partial o_{i,j}}}_{\lambda_{i,j}} \Big(\frac{\partial f_\theta(x_i)}{\partial \theta} - \frac{\partial f_\theta(x_i)}{\partial \theta} \Big)$$

Pairwise ranking loss

Scoring function itself

Current ranking list

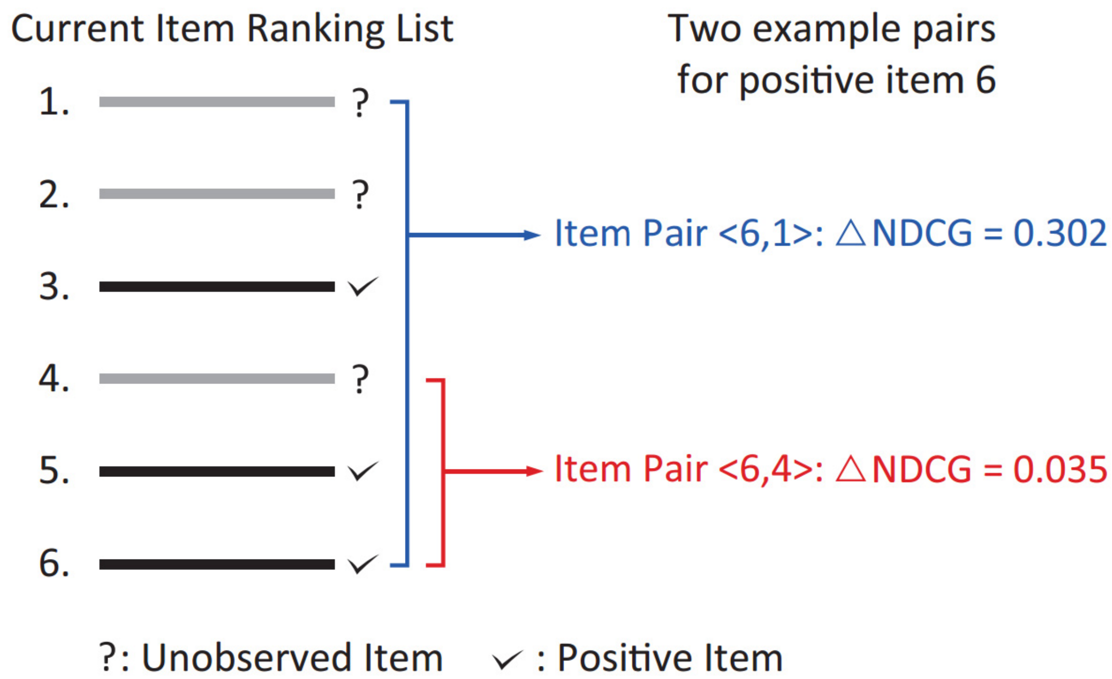
- LambdaRank basic idea
  - Add listwise information into $\lambda_{i,j}$ as $h(\lambda_{i,j}, g_q)$

$$\frac{\partial \mathcal{L}(q, d_i, d_j)}{\partial \theta} = h(\lambda_{i,j}, g_q) \Big(\frac{\partial f_\theta(x_i)}{\partial \theta} - \frac{\partial f_\theta(x_j)}{\partial \theta} \Big)$$

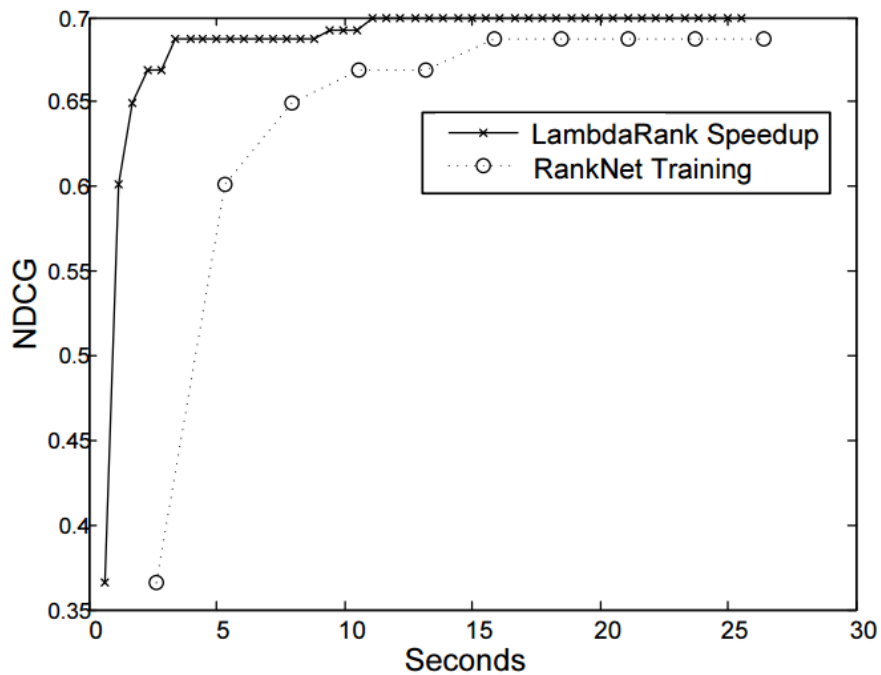
LambdaRank for Optimizing NDCG

- A choice of Lambda for optimizing NDCG

$$h(\lambda_{i,j}, g_q) = \lambda_{i,j} \Delta NDCG_{i,j}$$



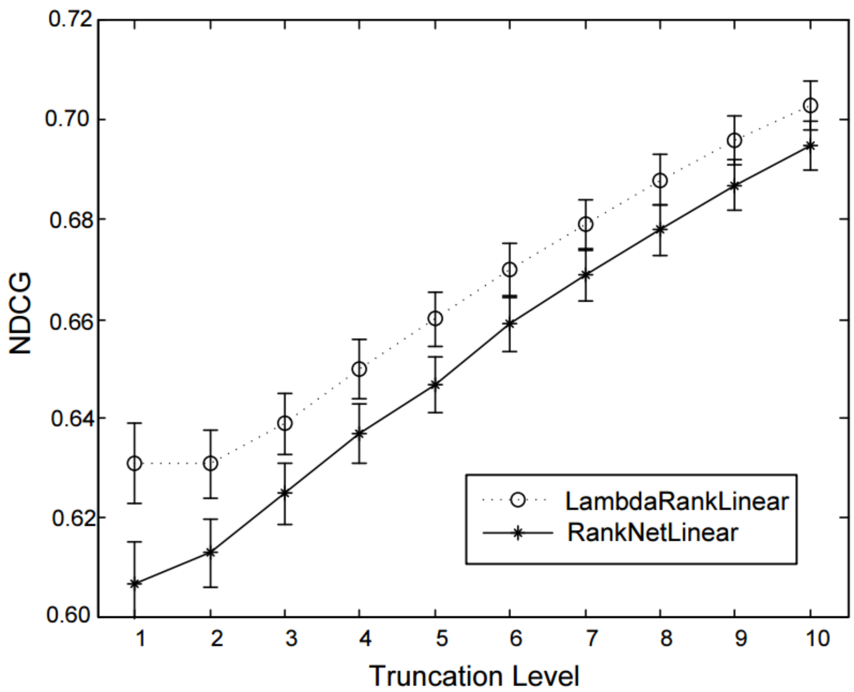
LambdaRank vs. RankNet

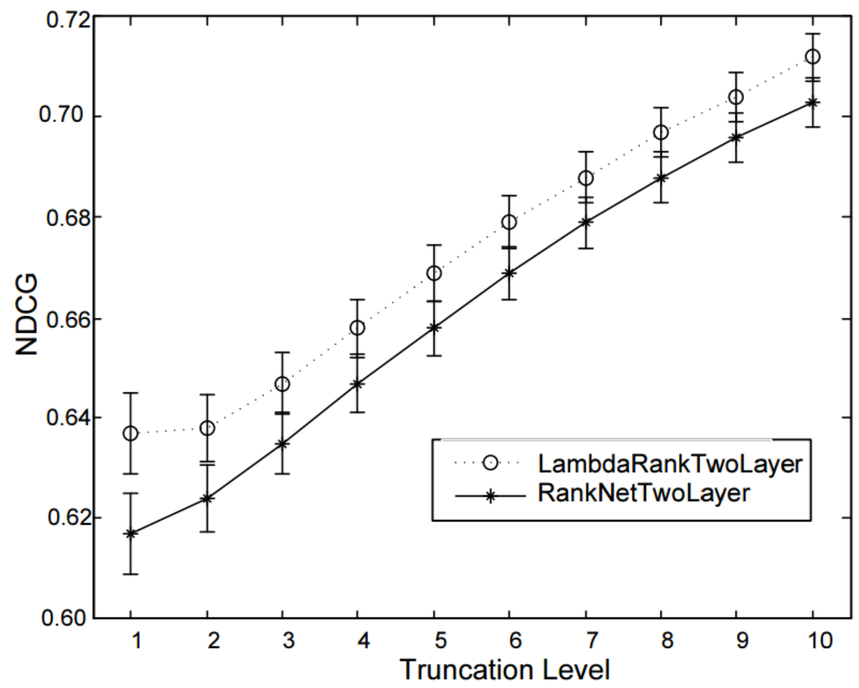


Linear nets

Burges, Christopher JC, Robert Ragno, and Quoc Viet Le. "Learning to rank with nonsmooth cost functions." *NIPS*. Vol. 6. 2006.

LambdaRank vs. RankNet





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Summary of Learning to Rank

- Pointwise, pairwise and listwise approaches for learning to rank
- Pairwise approaches are still the most popular
  - A balance of ranking effectiveness and training efficiency
- LambdaRank is a pairwise approach with list-level information
  - Easy to implement, easy to improve and adjust

A Data Mining Application: Recommendation

Overview

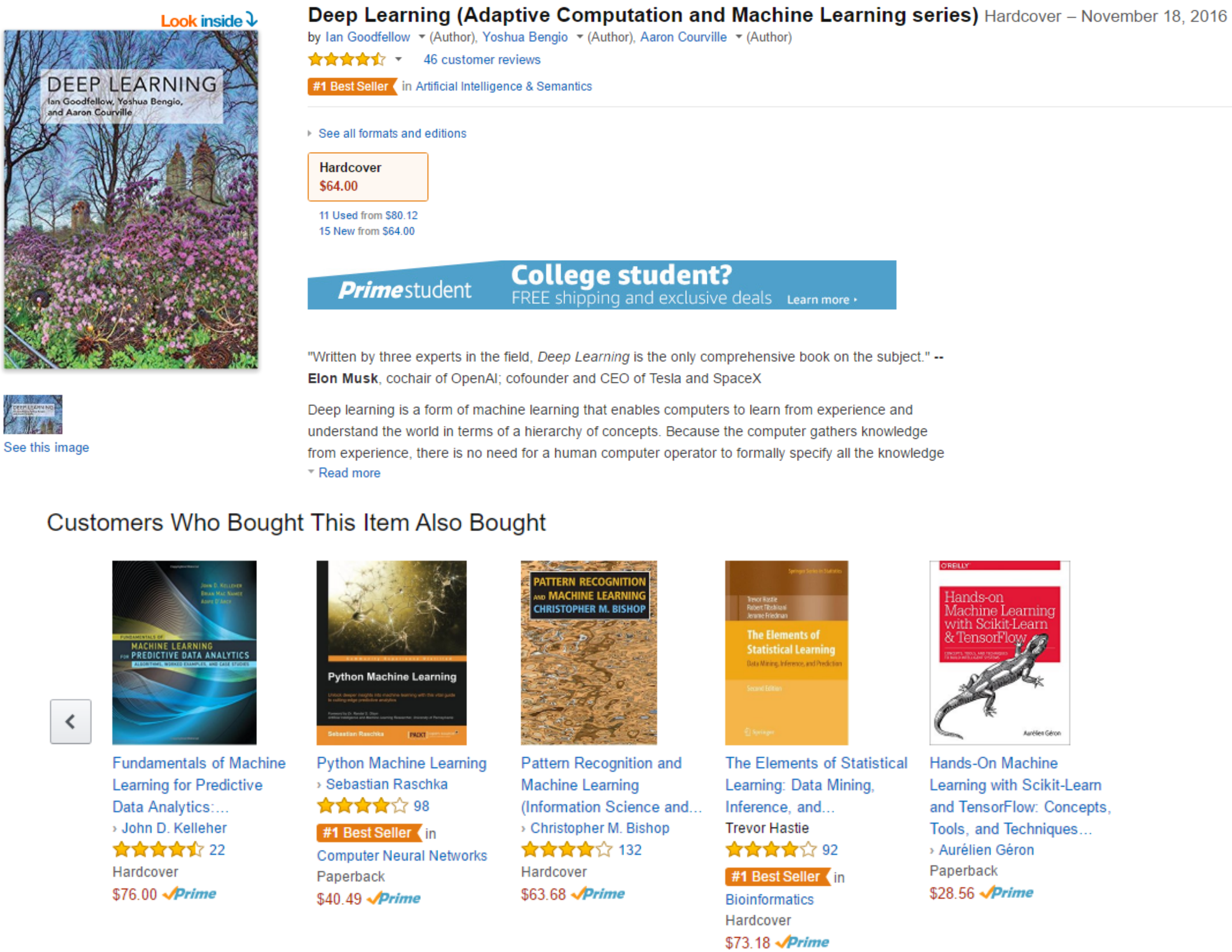
Collaborative Filtering

Rating prediction

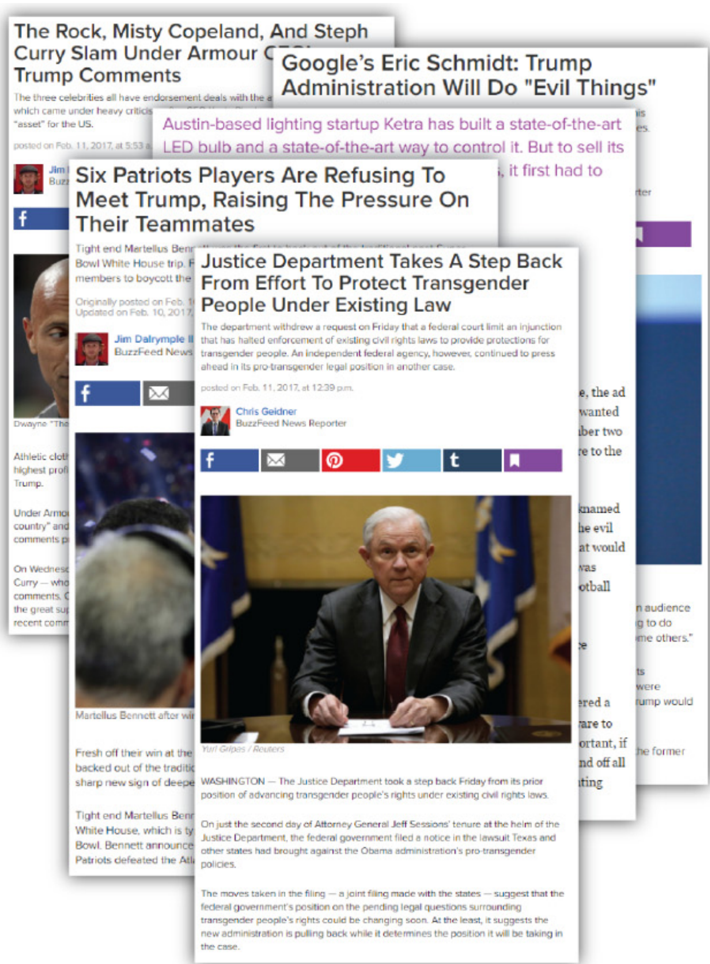
Top-N ranking

Sincerely thank Prof. Jun Wang

Book Recommendation

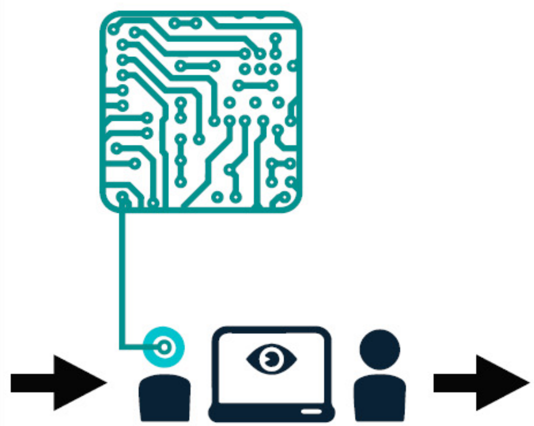


News Feed Recommendation

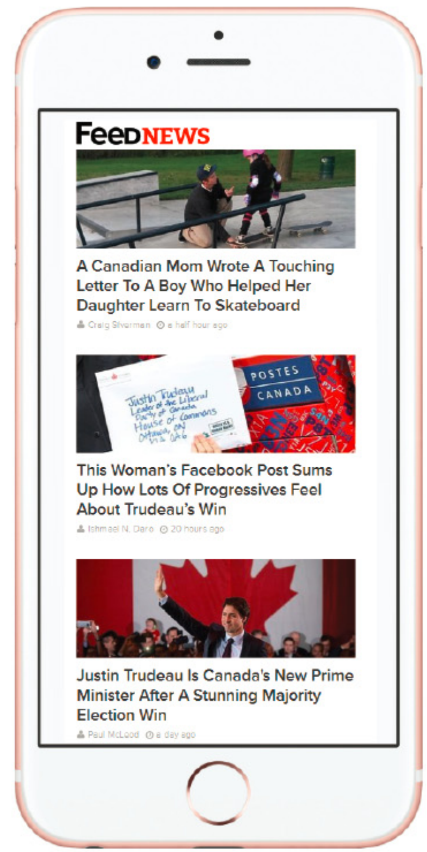


Huge numbers of candidate articles daily

Recommender System



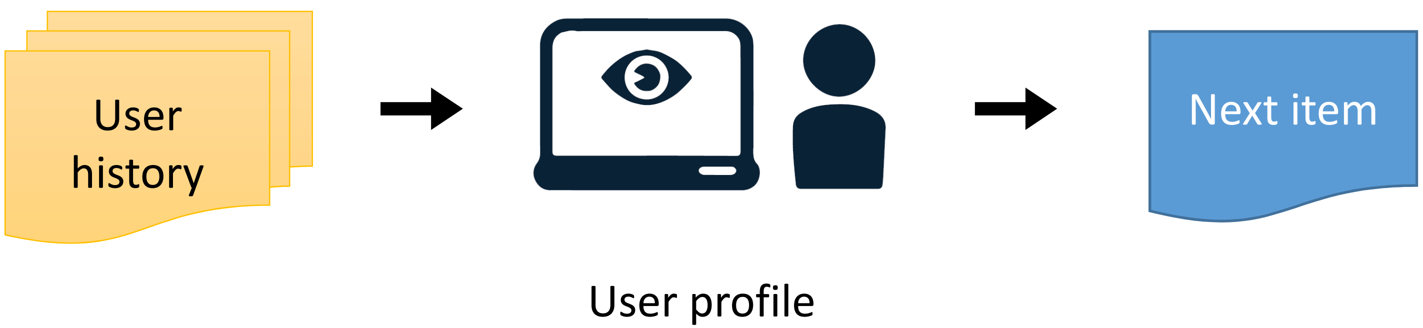
Editors manually select quality articles



Quality articles selected for news feed to end users

Personalized Recommendation

- Personalization framework



Build user profile from her history

- Ratings (e.g., amazon.com)
  - Explicit, but expensive to obtain
- Visits (e.g., newsfeed)
  - Implicit, but cheap to obtain

Personalization Methodologies

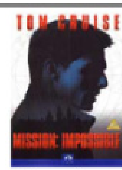
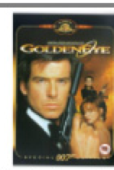
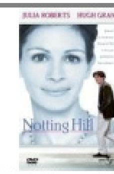
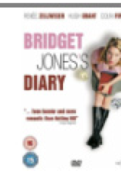
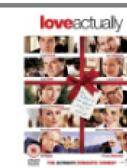
- Given the user's previous liked movies, how to recommend more movies she would like?
  - Method 1: recommend the movies that share the actors/actresses/director/genre with those the user likes
  - Method 2: recommend the movies that the users with similar interest to her like

Information Filtering

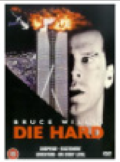
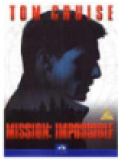
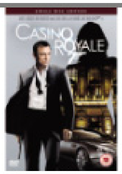
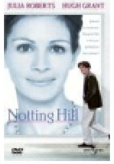
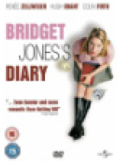
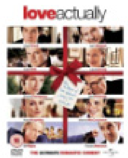
- Information filtering deals with the delivery of information that the user is likely to find interesting or useful
  - Recommender system: information filtering in the form of suggestions
  - Two approaches for information filtering
    - Content-based filtering
      - recommend the movies that share the actors/actresses/director/genre with those the user likes
    - Collaborative filtering (the focus of this lecture)
      - recommend the movies that the users with similar interest to her like

http://recommender-systems.org/information-filtering/

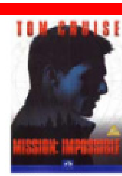
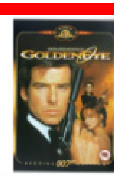
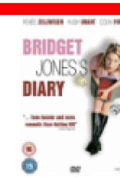
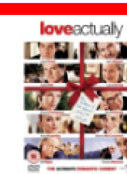
A (small) Rating Matrix

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

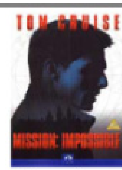
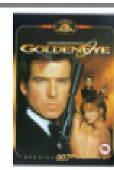
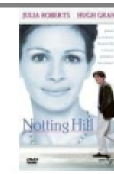
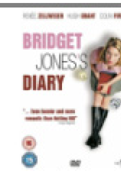
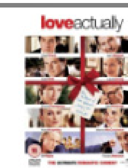
The Users

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

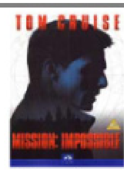
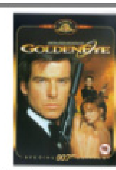
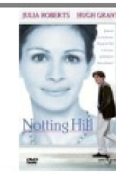
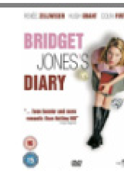
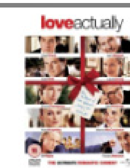
The Items

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

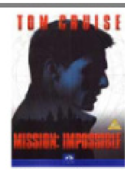
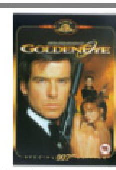
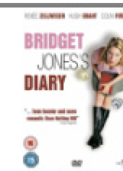
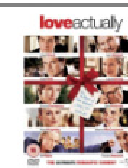
A User-Item Rating

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

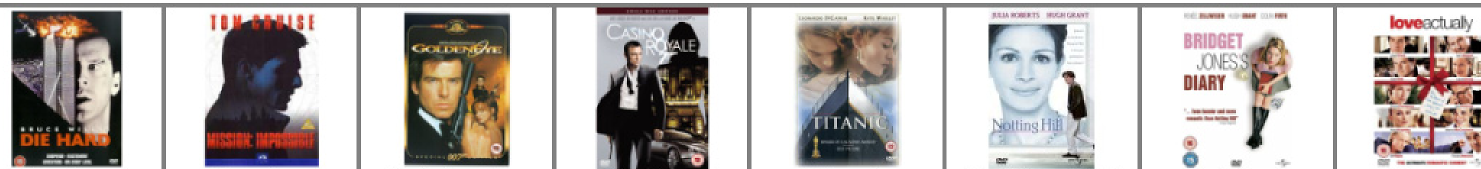
A User Profile

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

An Item Profile

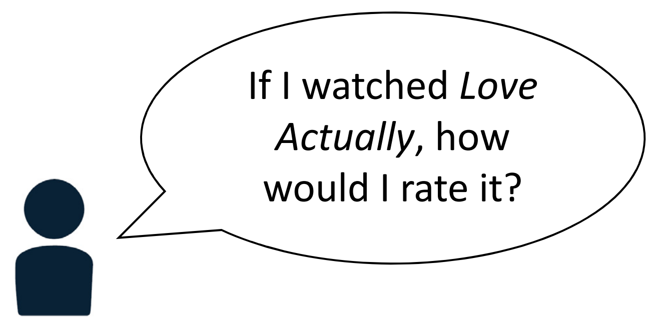
	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

A Null Rating Entry



	Die Hard	Mission: Impossible	GoldenEye	Casino Royale	Titanic	Notting Hill	Bridget Jones's Diary	Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

- Recommendation on explicit data
  - Predict the null ratings



K Nearest Neighbor Algorithm (KNN)

- A non-parametric method used for classification and regression

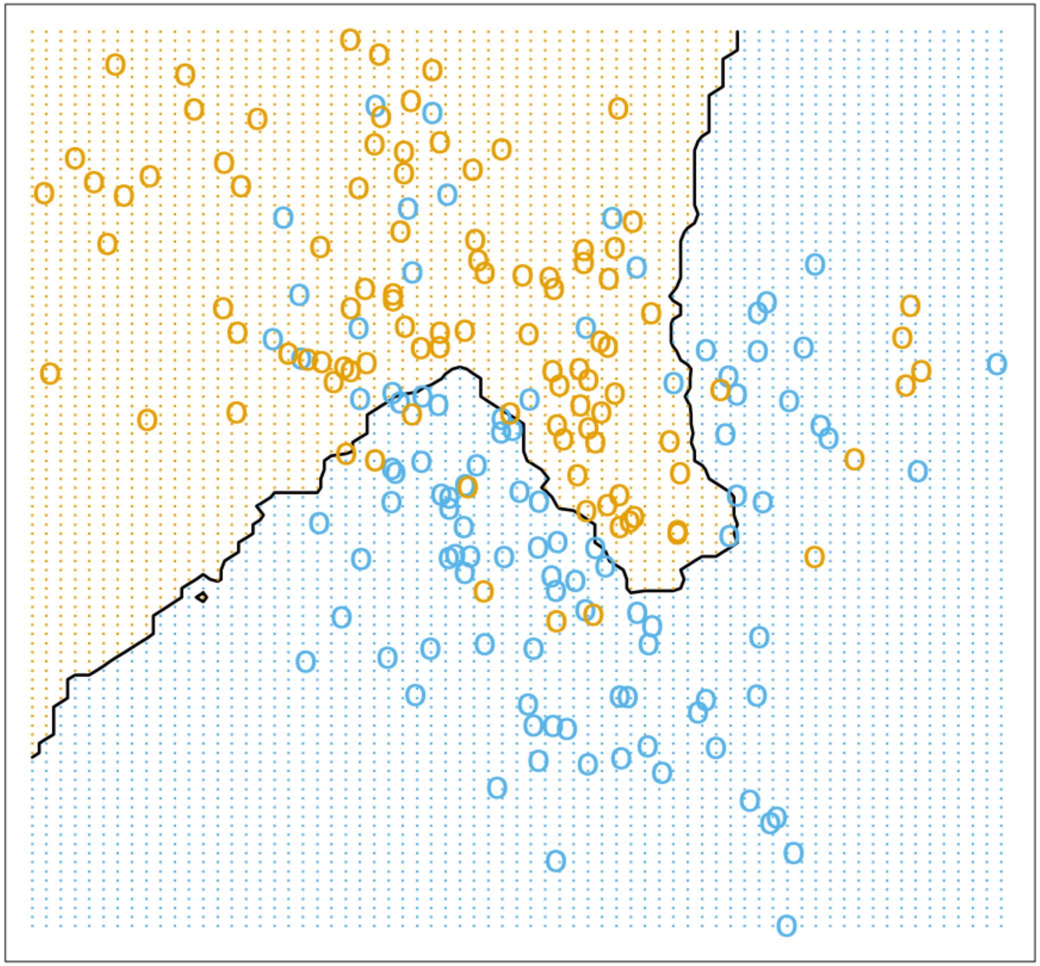
  - for each input instance *x*, find *k* closest training instances $N_k(x)$ in the feature space
  - the prediction of *x* is based on the average of labels of the *k* instances

$$\hat{y}(x) = \frac{1}{k} \sum_{x_i \in N_k(x)} y_i$$

  - For classification problem, it is the majority voting among neighbors

kNN Example

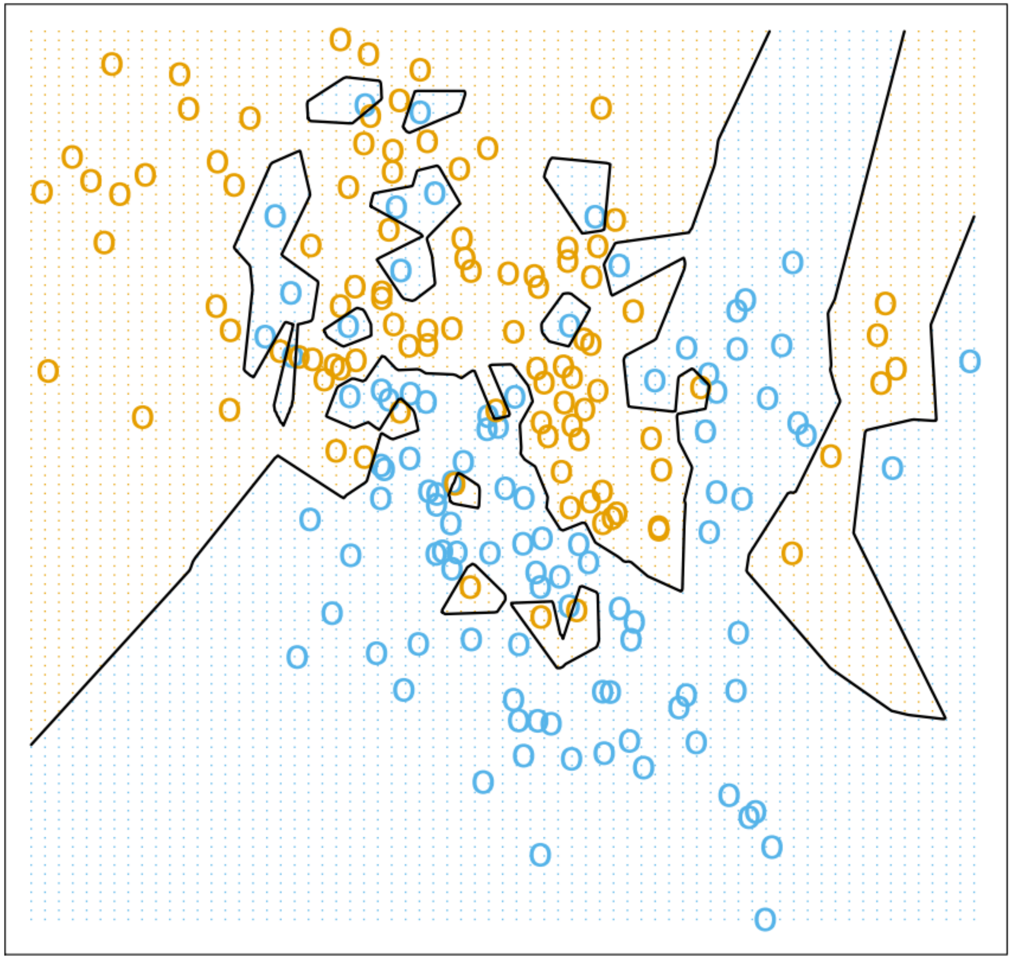
15-nearest neighbor



Jerome H. Friedman, Robert Tibshirani, and Trevor Hastie. “The Elements of Statistical Learning”. Springer 2009.

kNN Example

1-nearest neighbor



Jerome H. Friedman, Robert Tibshirani, and Trevor Hastie. “The Elements of Statistical Learning”. Springer 2009.

K Nearest Neighbor Algorithm (KNN)

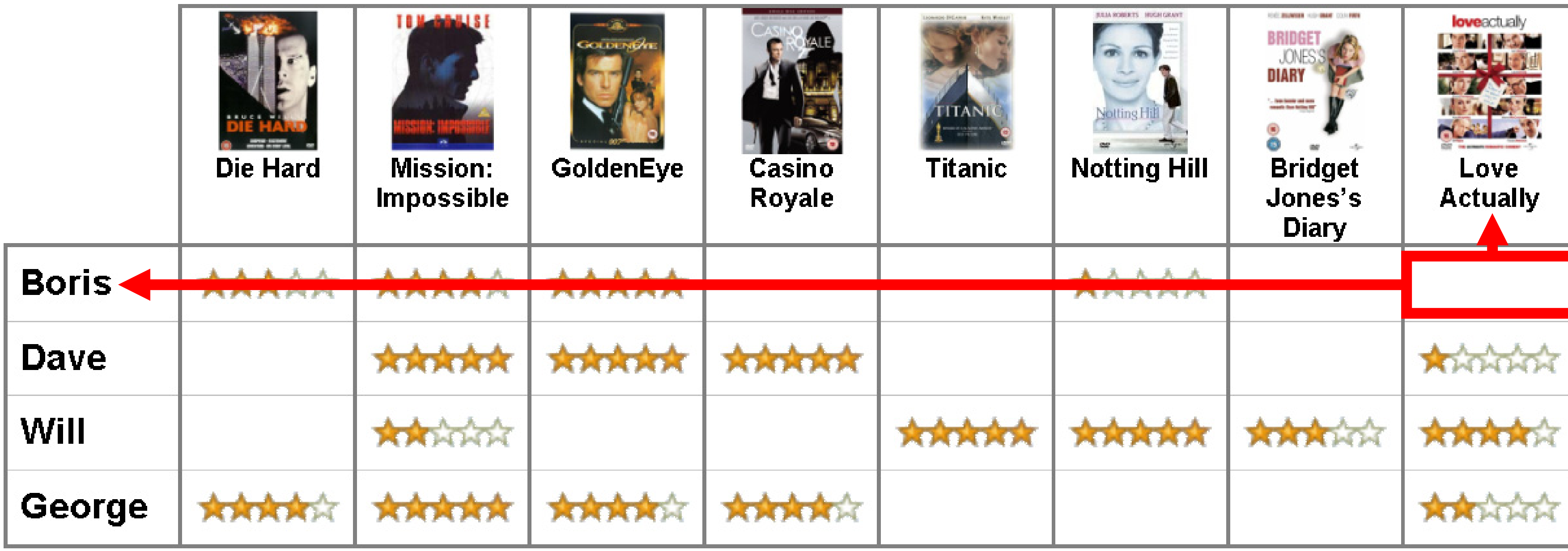
- Generalized version
  - Define similarity function $s(x, x_i)$ between the input instance x and its neighbor x_i
  - Then the prediction is based on the weighted average of the neighbor labels based on the similarities

$$\hat{y}(x) = \frac{\sum_{x_i \in N_k(x)} s(x, x_i) y_i}{\sum_{x_i \in N_k(x)} s(x, x_i)}$$

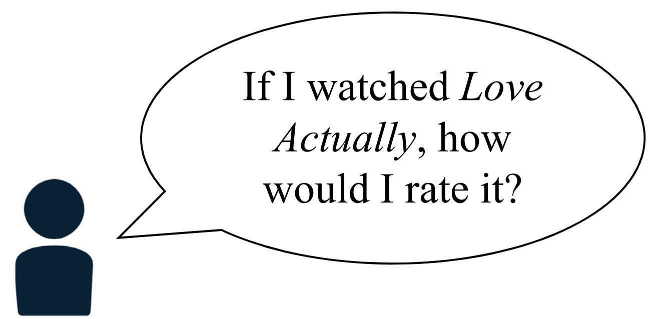
Non-Parametric kNN

- No parameter to learn
  - In fact, there are N parameters: each instance is a parameter
  - There are N/k effective parameters
    - Intuition: if the neighborhoods are non-overlapping, there would be N/k neighborhoods, each of which fits one parameter
- Hyperparameter k
  - We cannot use sum-of-squared error on the training set as a criterion for picking k, since k=1 is always the best
  - Tune k on validation set

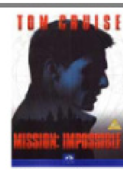
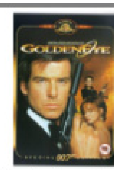
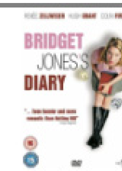
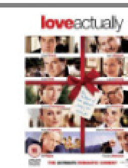
A Null Rating Entry



- Recommendation on explicit data
  - Predict the null ratings

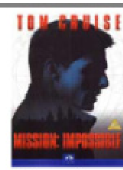
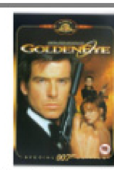
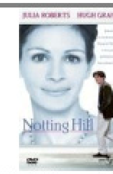
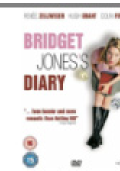
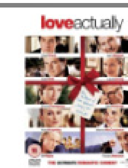


Collaborative Filtering Example

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		**?**
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

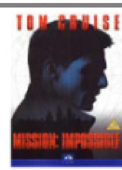
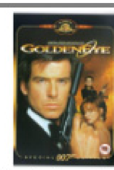
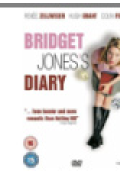
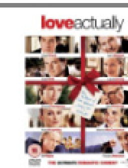
- What do you think the rating would be?

User-based kNN Solution

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
▶ Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		?
▶ Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
▶ George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

- Find similar users (neighbors) for Boris
  - Dave and George

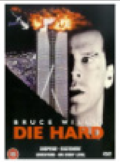
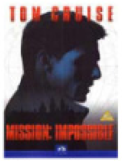
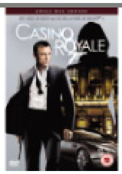
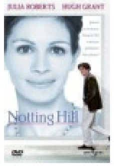
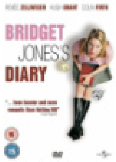
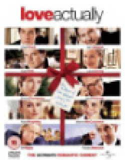
Rating Prediction

	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		★½☆☆☆
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

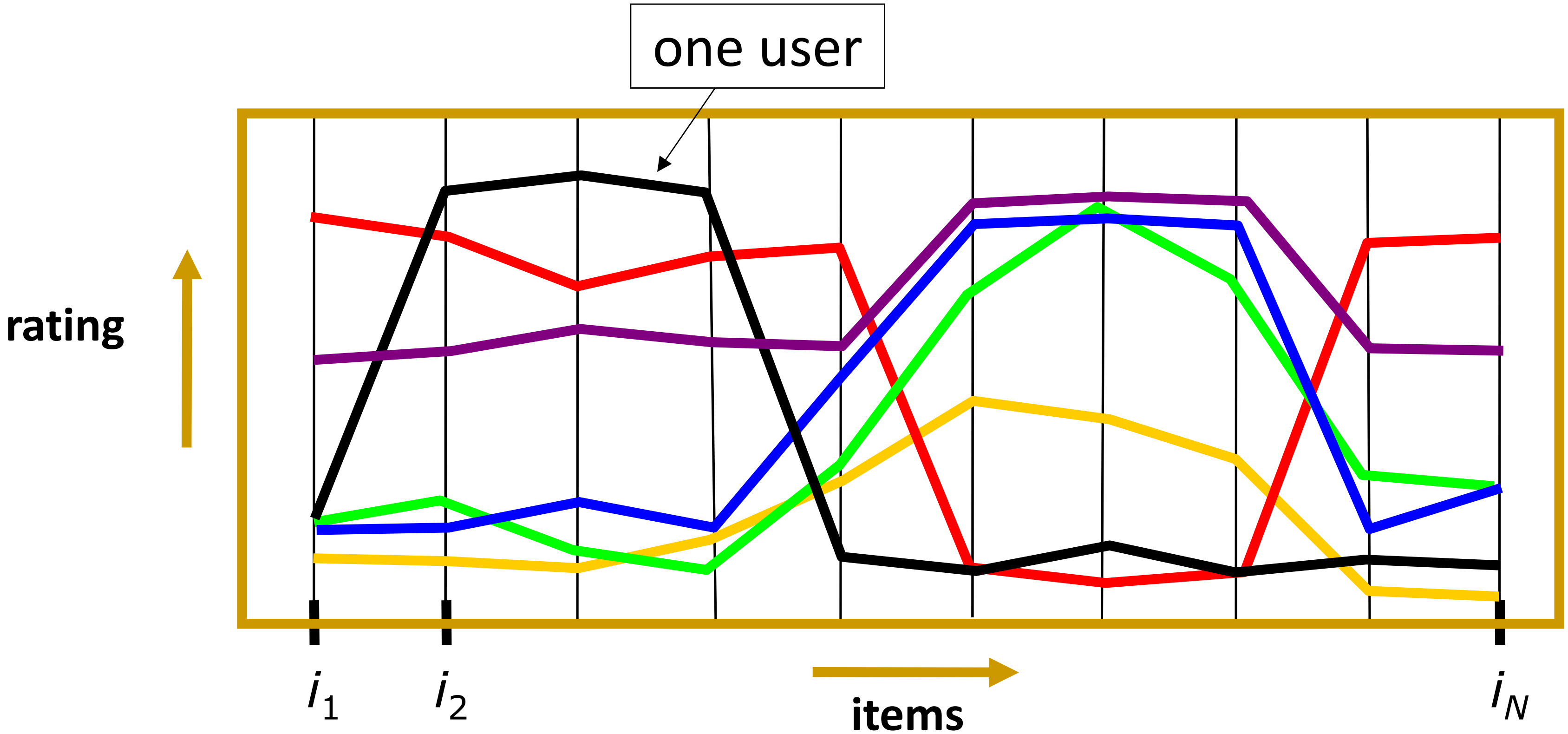
- Average Dave's and George's rating on Love Actually
  - Prediction = (1+2)/2 = 1.5

Collaborative Filtering for Recommendation

- Basic user-based kNN algorithm
  - For each target user for recommendation
    1. Find similar users
    2. Based on similar users, recommend new items

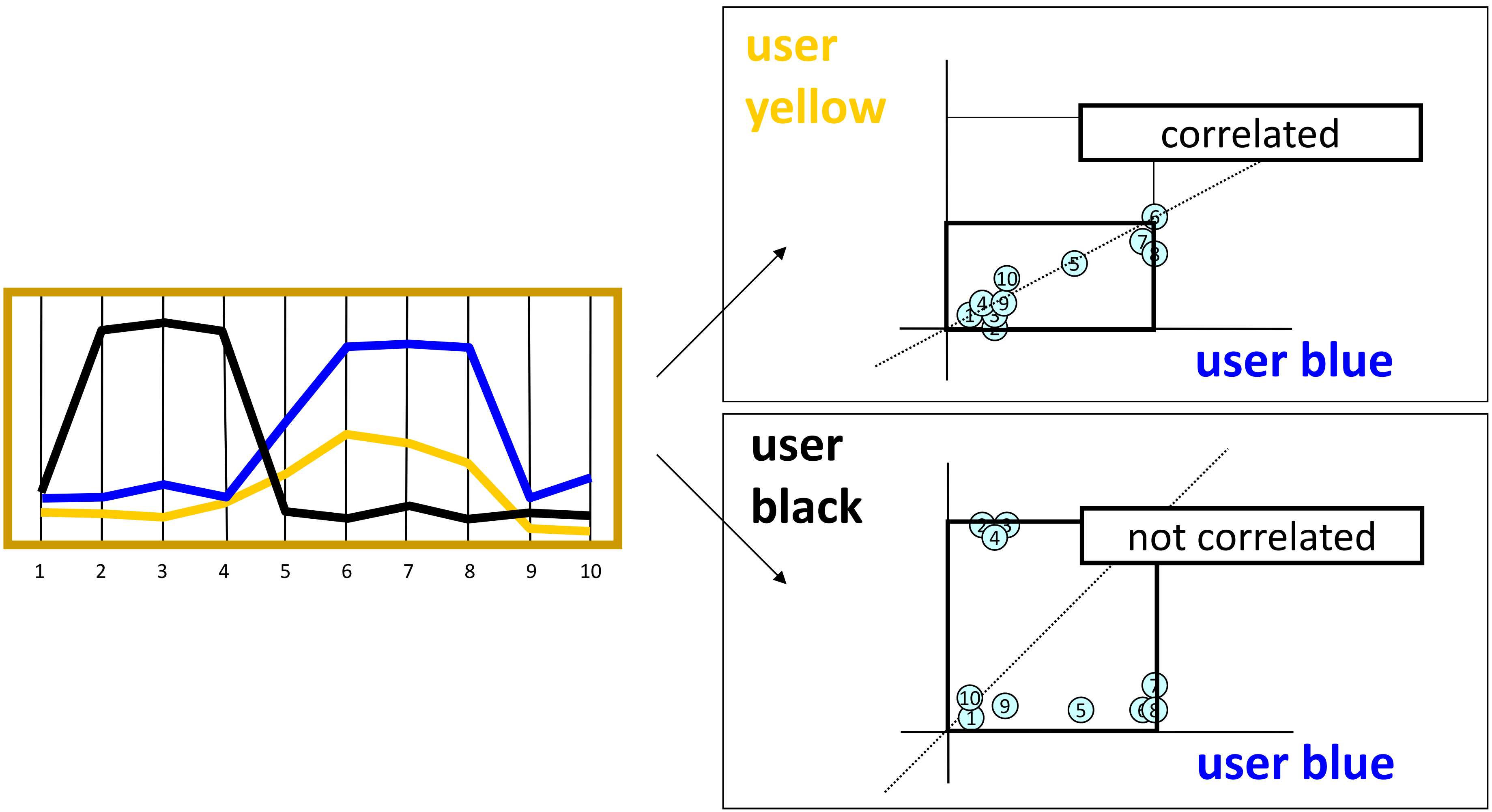
	 Die Hard	 Mission: Impossible	 GoldenEye	 Casino Royale	 Titanic	 Notting Hill	 Bridget Jones's Diary	 Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		★★☆☆☆
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

Similarity between Users

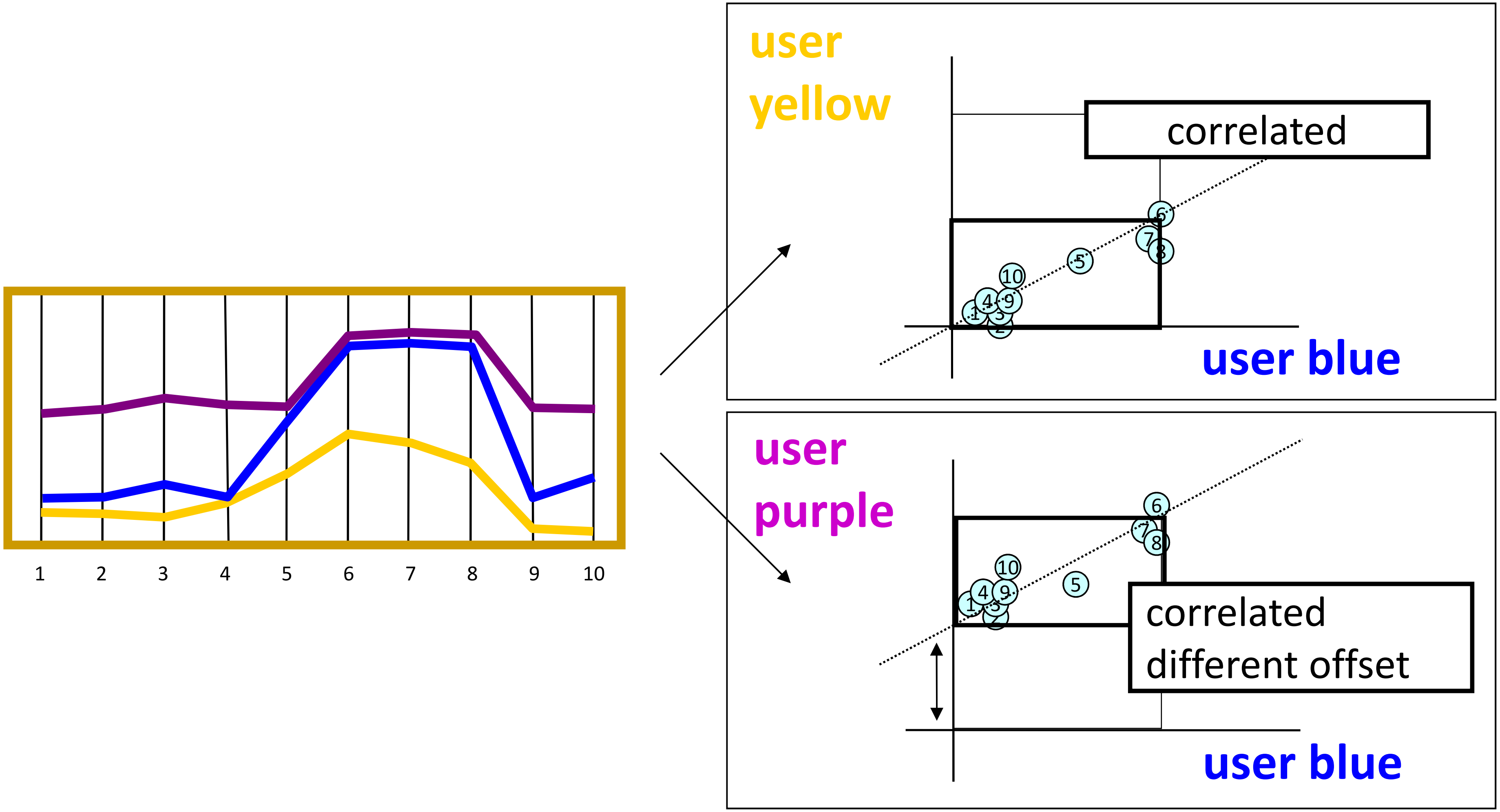


- Each user's profile can be directly built as a vector based on her item ratings

Similarity between Users



Similarity between Users



Similarity Measures (Users)

- Similarity measures between two users *a* and *b*
  - Cosine (angle)

$$s_u^{\cos}(u_a, u_b) = \frac{u_a^\top u_b}{\|u_a\| \|u_b\|} = \frac{\sum_m x_{a,m} x_{b,m}}{\sqrt{\sum_m x_{a,m}^2 \sum_m x_{b,m}^2}}$$

  - Pearson Correlation

$$s_u^{\mathrm{corr}}(u_a, u_b) = \frac{\sum_m (x_{a,m} - \bar{x}_a)(x_{b,m} - \bar{x}_b)}{\sqrt{\sum_m (x_{a,m} - \bar{x}_a)^2 \sum_m (x_{b,m} - \bar{x}_b)^2}}$$

User-based kNN Rating Prediction

- Predicting the rating from target user *t* to item *m*

Weights according to similarity with query user

predict according to rating of similar user (corrected for average rating of that user)

$$\hat{x}_{t,m} = \bar{x}_t + \frac{\sum_{u_a \in N_u(u_t)} s_u(u_t, u_a)(x_{a,m} - \bar{x}_a)}{\sum_{u_a \in N_u(u_t)} s_u(u_t, u_a)}$$

correct for average rating query user

$$\bar{u}_t = \frac{1}{|I(u_t)|} \sum_{m \in I(u_t)} x_{t,m}$$

normalize such that ratings stay between 0 and *R*

Item-based kNN Solution

- Recommendation based on item similarity

query item

similar items based on profiles

	i_1	i_2	i_3	i_4	i_5	…	i_M
u_1	2	0	3	2	5	…	1
u_2	0	4	0	0	0	…	5
u_3	0	2	0	0	0	…	4
u_4	1	0	4	2	4	…	2
…	…	…	…	…	…	…	…
u_K	2	0	$\boxed{4}$	?	$\boxed{4}$	…	1

target user

predicted rating: 4

Item-based kNN Solution

- For each unrated items *m* of the target user *t*
  - Find similar items {*a*}
  - Based on the set of similar items {*a*}
    - Predict the rating of the item *m*

query item

similar items based on profiles

	i_1	i_2	i_3	i_4	i_5	...	i_M
u_1	2	0	3	2	5	...	1
u_2	0	4	0	0	0	...	5
u_3	0	2	0	0	0	...	4
u_4	1	0	4	2	4	...	2
...	...	...	...	...	...	...	...
u_K	2	0	$\boxed{4}$	?	$\boxed{4}$	...	1

target user

predicted rating: 4

Similarity Measures (Items)

- Similarity measures between two items *a* and *b*
  - Cosine (angle)

$$s_i^{\cos}(i_a, i_b) = \frac{i_a^\top i_b}{\|i_a\| \|i_b\|} = \frac{\sum_u x_{u,a} x_{u,b}}{\sqrt{\sum_u x_{u,a}^2 \sum_u x_{u,b}^2}}$$

  - Adjusted Cosine

$$s_i^{\mathrm{adcos}}(i_a, i_b) = \frac{\sum_u (x_{u,a} - \bar{x}_u)(x_{u,b} - \bar{x}_u)}{\sqrt{\sum_u (x_{u,a} - \bar{x}_u)^2 \sum_u (x_{u,b} - \bar{x}_u)^2}}$$

  - Pearson Correlation

$$s_i^{\mathrm{corr}}(i_a, i_b) = \frac{\sum_u (x_{u,a} - \bar{x}_a)(x_{u,b} - \bar{x}_b)}{\sqrt{\sum_u (x_{u,a} - \bar{x}_a)^2 \sum_u (x_{u,b} - \bar{x}_b)^2}}$$

Item-based kNN Rating Prediction

- Get top-*k* neighbor items that the target user *t* rated

Rank position

$$N_i(u_t, i_a) = \{i_b | r_i(i_a, i_b) < K^*, x_{t,b} \neq 0\}$$

Choose *K** such that $|N_i(u_t, i_a)| = k$

- Predict ratings for item *a* that the target user *t* did not rate

$$\hat{x}_{t,a} = \frac{\sum_{i_b \in N_i(u_t, i_a)} s_i(i_a, i_b) x_{t,b}}{\sum_{i_b \in N_i(u_t, i_a)} s_i(i_a, i_b)}$$

Don't need to correct for users average rating since query user itself is used to do predictions

Empirical Study

- Movielens dataset from grouplens
  - http://www.grouplens.org/node/73
  - Users visit Movielens
    - rate and receive recommendations for movies
  - Dataset (ML-100k)
    - 100k ratings from 1 to 5
    - 943 users, 1682 movies (rated by at least one user)
    - Sparsity level

$$1 - \frac{\#\text{non-zero entries}}{\text{total entries}} = 1 - \frac{10^5}{943 \times 1682} = 93.69\%$$

Experiment Setup

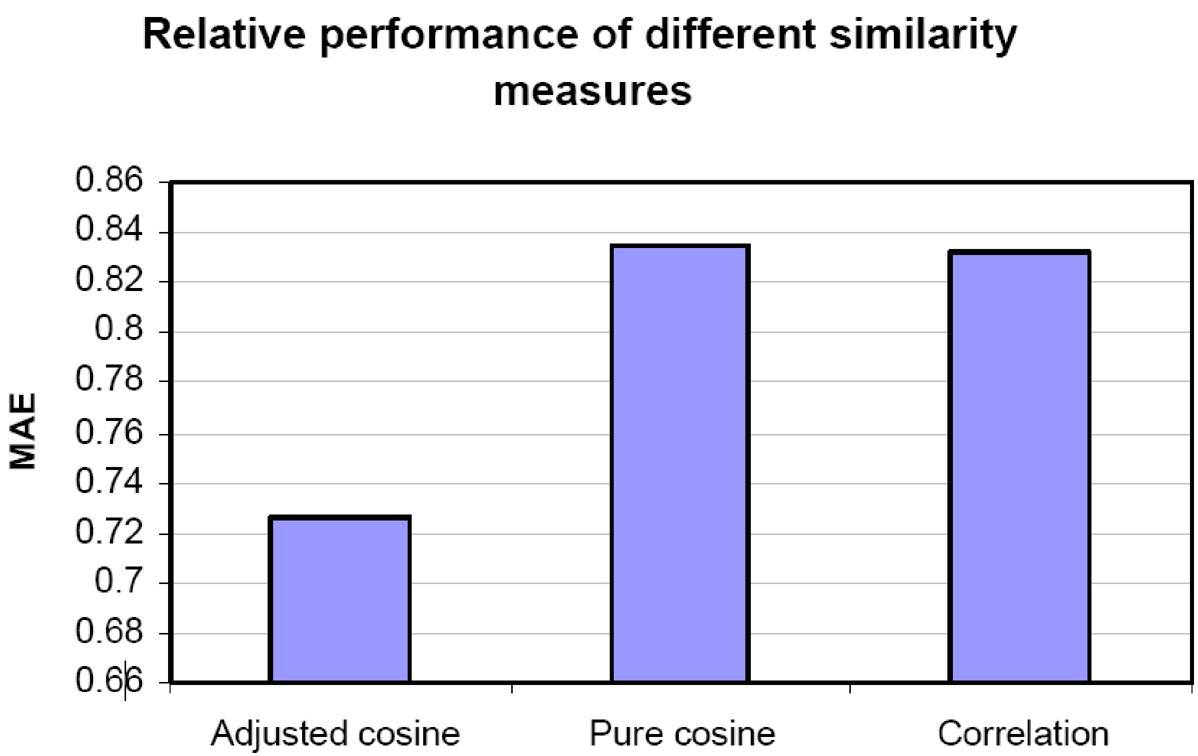
- Split data in training (*x*%) and test set ((100-*x*)%)
  - Can be repeated *T* times and results averaged
- Evaluation metrics
  - Mean-Absolute Error (MAE)

$$\mathrm{MAE} = \frac{1}{|D_{\mathrm{test}}|} \sum_{(u,i,r) \in D_{\mathrm{test}}} |r - \hat{r}_{u,i}|$$

  - Root Mean Squared Error (RMSE)

$$\mathrm{RMSE} = \sqrt{\frac{1}{|D_{\mathrm{test}}|} \sum_{(u,i,r) \in D_{\mathrm{test}}} (r - \hat{r}_{u,i})^2}$$

Impact of Similarity Measures

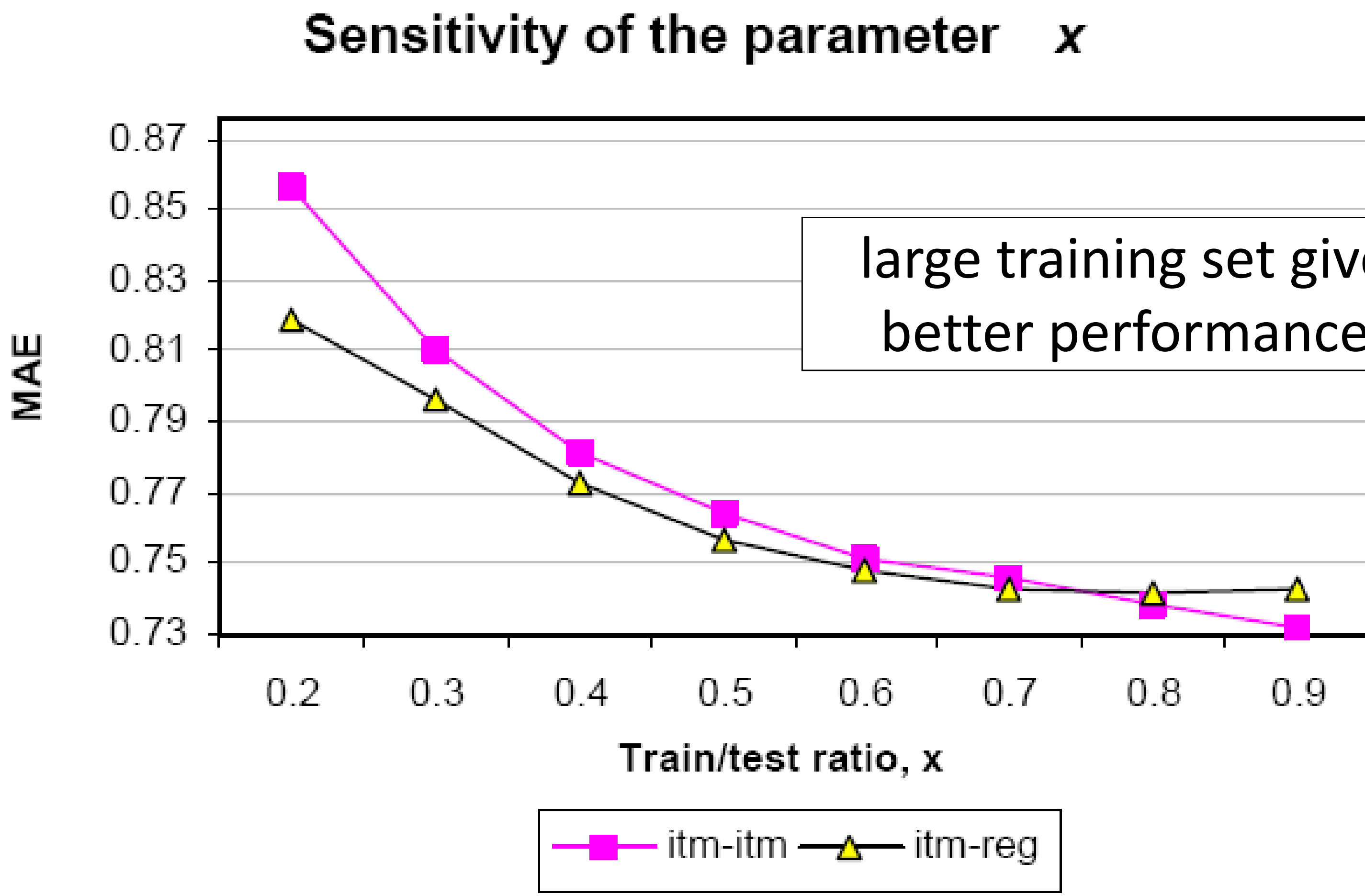


$$s_i^{\mathrm{adcos}}(i_a, i_b) = \frac{\sum_u (x_{u,a} - \bar{x}_u)(x_{u,b} - \bar{x}_u)}{\sqrt{\sum_u (x_{u,a} - \bar{x}_u)^2 \sum_u (x_{u,b} - \bar{x}_u)^2}}$$

$$s_i^{\mathrm{corr}}(i_a, i_b) = \frac{\sum_u (x_{u,a} - \bar{x}_a)(x_{u,b} - \bar{x}_b)}{\sqrt{\sum_u (x_{u,a} - \bar{x}_a)^2 \sum_u (x_{u,b} - \bar{x}_b)^2}}$$

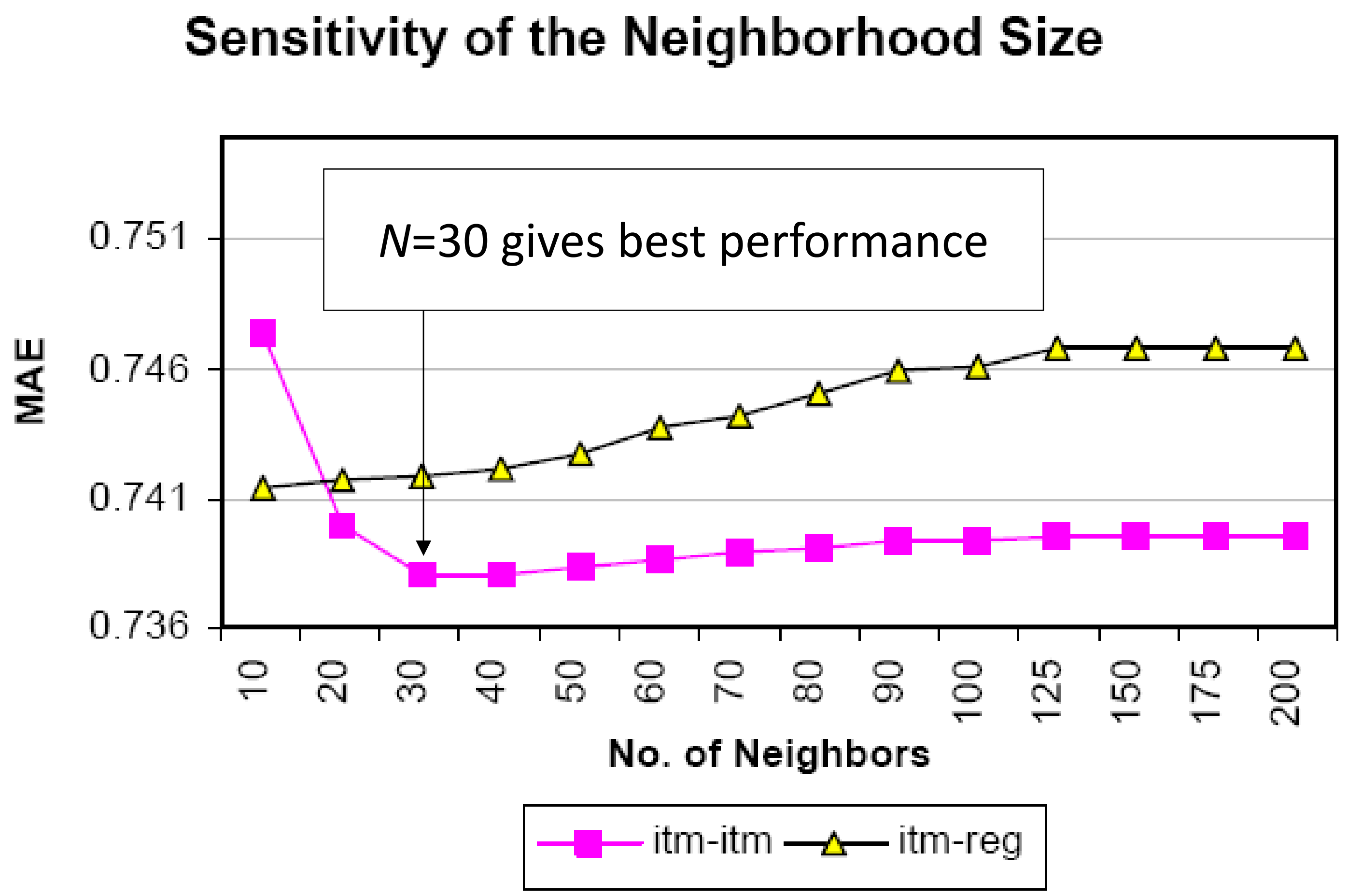
$$s_i^{\cos}(i_a, i_b) = \frac{i_a^\top i_b}{\|i_a\| \|i_b\|} = \frac{\sum_u x_{u,a} x_{u,b}}{\sqrt{\sum_u x_{u,a}^2 \sum_u x_{u,b}^2}}$$

Sensitivity of Train/Test Ratio

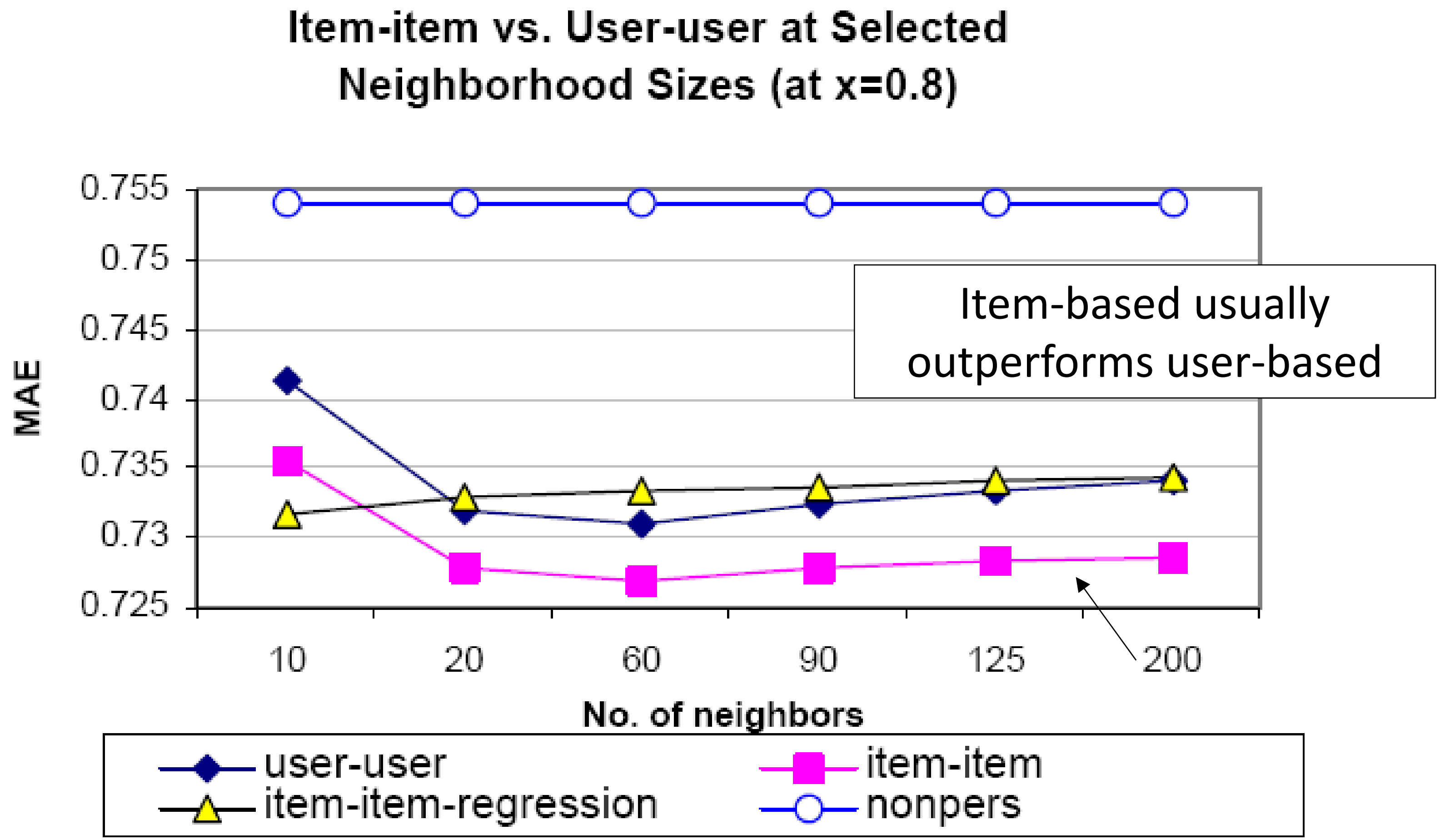


large training set gives better performances

Sensitivity Neighborhood Size *k*



Item-based vs. User-based



- Item-item similarity is usually more stable and objective

kNN based Methods Summary

- Straightforward and highly explainable
- No parameter learning
  - Only one hyperparameter *k* to tune
  - Cannot get improved by learning
- Efficiency could be a serious problem
  - When the user/item numbers are large
  - When there are a huge number of user-item ratings
- We may need a parametric and learnable model

Matrix Factorization Techniques

	Die Hard	Mission: Impossible	GoldenEye	Casino Royale	Titanic	Notting Hill	Bridget Jones's Diary	Love Actually
Boris	★★★☆☆	★★★★☆	★★★★★			★☆☆☆☆		★½☆☆☆
Dave		★★★★★	★★★★★	★★★★★				★☆☆☆☆
Will		★★☆☆☆			★★★★★	★★★★★	★★★☆☆	★★★★☆
George	★★★★☆	★★★★★	★★★★☆	★★★★☆				★★☆☆☆

$\simeq$

Koren, Yehuda, Robert Bell, and Chris Volinsky. "Matrix factorization techniques for recommender systems." *Computer* 42.8 (2009).

Matrix Factorization Techniques

Items

Users

1		3			5			5		4	
		5	4	?		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

$$\hat{r}_{u,i} = p_u^\top q_i$$

$\simeq$

Users

	.1	-.4	.2
u	-.5	.6	.5
	-.2	.3	.5
	1.1	2.1	.3
	-.7	2.1	-2
	-1	.7	.3

$\cdot$

Items

				i							
1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

Basic MF Model

- Prediction of user *u*'s rating on item *i*

$$\hat{r}_{u,i} = p_u^\top q_i$$

Bilinear model

- Loss function

$$\mathcal{L}(u, i, r_{u,i}) = \frac{1}{2}(r_{u,i} - p_u^\top q_i)^2$$

- Training objective

$$\min_{P,Q} \sum_{r_{u,i} \in D} \frac{1}{2}(r_{u,i} - p_u^\top q_i)^2 + \frac{\lambda}{2}(\|p_u\|^2 + \|q_i\|^2)$$

- Gradients

$$\frac{\partial \mathcal{L}(u, i, r_{u,i})}{\partial p_u} = (p_u^\top q_i - r_{u,i})q_i + \lambda p_u$$

$$\frac{\partial \mathcal{L}(u, i, r_{u,i})}{\partial q_i} = (p_u^\top q_i - r_{u,i})p_u + \lambda q_i$$

MF with Biases

- Prediction of user *u*'s rating on item *i*

$$\hat{r}_{u,i} = \underbrace{\mu}_{\text{Global bias}} + \underbrace{b_u}_{\text{User bias}} + \underbrace{b_i}_{\text{Item bias}} + \underbrace{p_u^\top q_i}_{\text{User-item Interaction}}$$

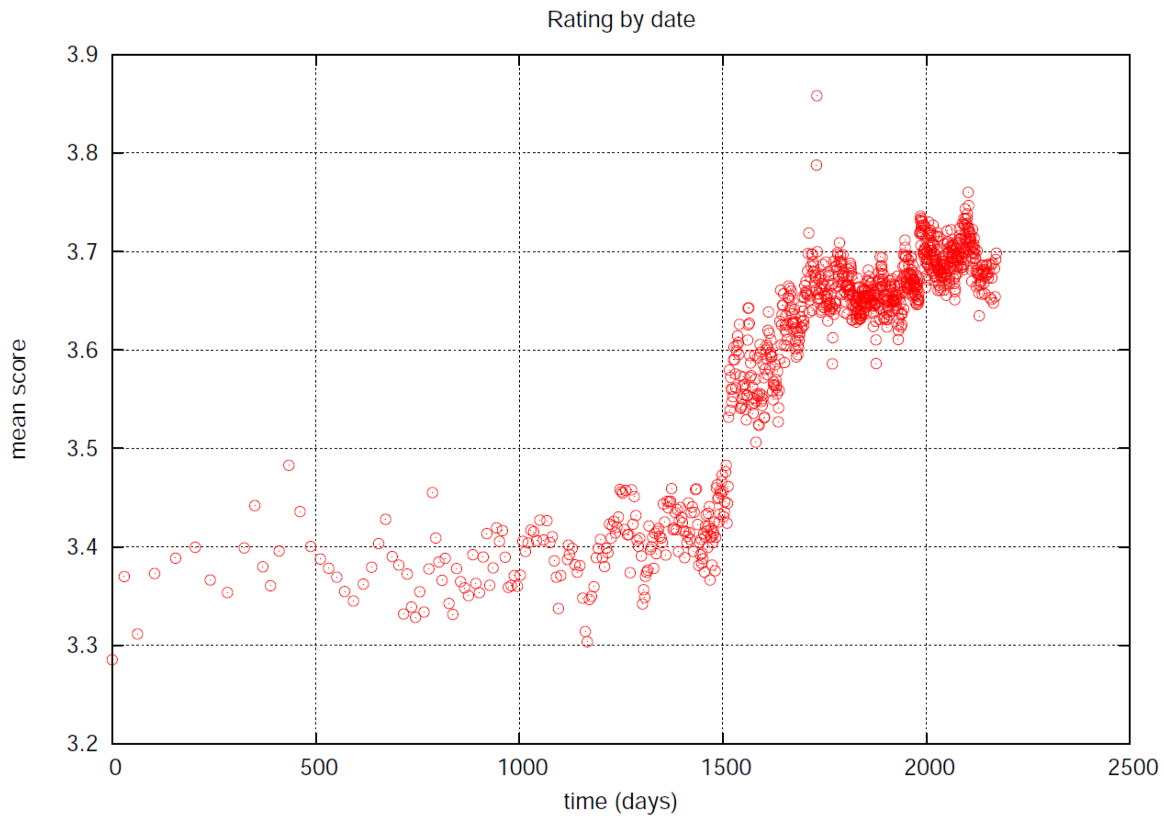
- Training objective

$$\min_{P,Q} \sum_{r_{u,i} \in D} \frac{1}{2}\Big(r_{u,i} - (\mu + b_u + b_i + p_u^\top q_i)\Big)^2 + \frac{\lambda}{2}(\|p_u\|^2 + \|q_i\|^2 + b_u^2 + b_i^2)$$

- Gradient update

$$\begin{aligned}
\delta &= r_{u,i} - (\mu + b_u + b_i + p_u^\top q_i) \\
\mu &\leftarrow \mu + \eta\delta \\
b_u &\leftarrow (1 - \eta\lambda)b_u + \eta\delta \\
b_i &\leftarrow (1 - \eta\lambda)b_i + \eta\delta \\
p_u &\leftarrow (1 - \eta\lambda)p_u + \eta\delta q_i \\
q_i &\leftarrow (1 - \eta\lambda)q_i + \eta\delta p_u
\end{aligned}$$

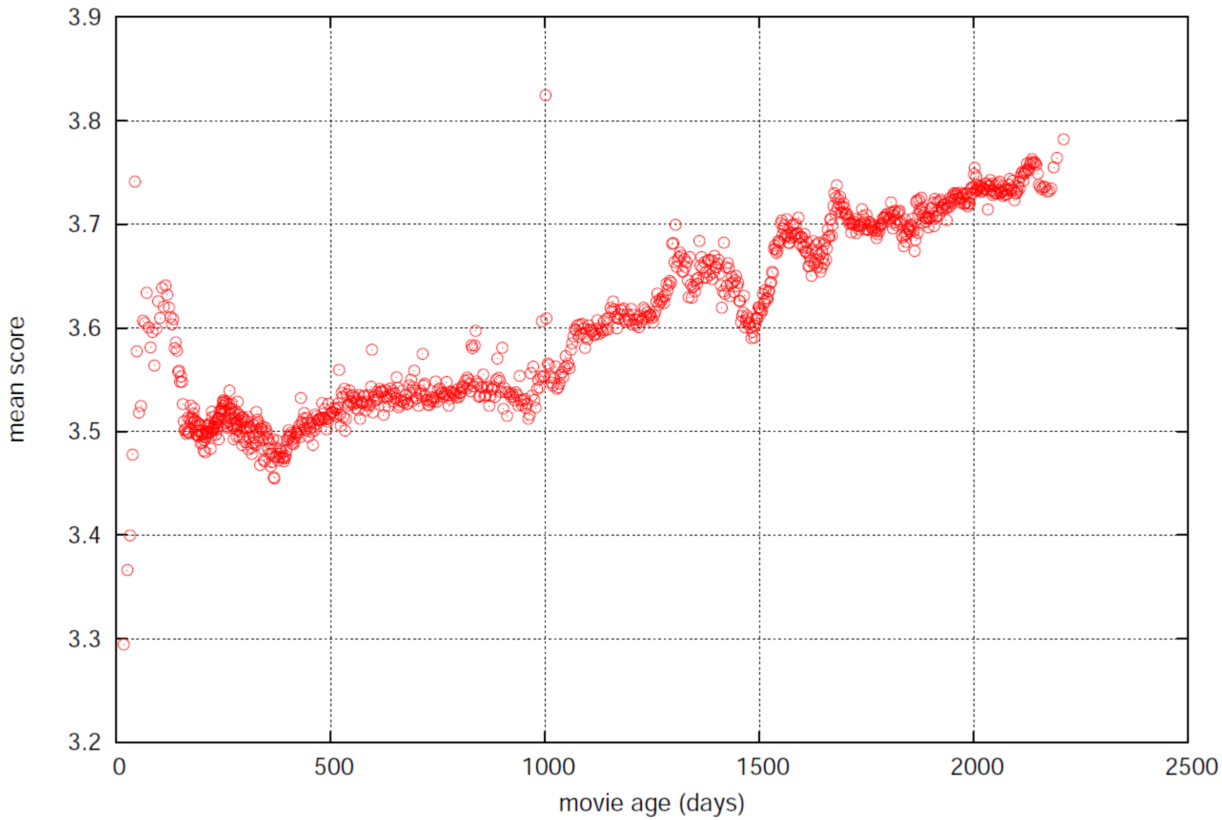
Temporal Dynamics



- A sudden rise in the average movie rating begging around 1500 days (early 2004) into the dataset

Koren, Yehuda. "Collaborative filtering with temporal dynamics." *KDD 2009*.

Temporal Dynamics



- People tend to give higher ratings as movies become older

Koren, Yehuda. "Collaborative filtering with temporal dynamics." *KDD 2009*.

Multiple sources of temporal dynamics

- Item-side effects
  - Product perception and popularity are constantly changing
  - Seasonal patterns influence items’ popularity

- User-side effects
  - Customers ever redefine their taste
  - Transient, short-term bias
  - Drifting rating scale
  - Change of rater within household

Addressing temporal dynamics

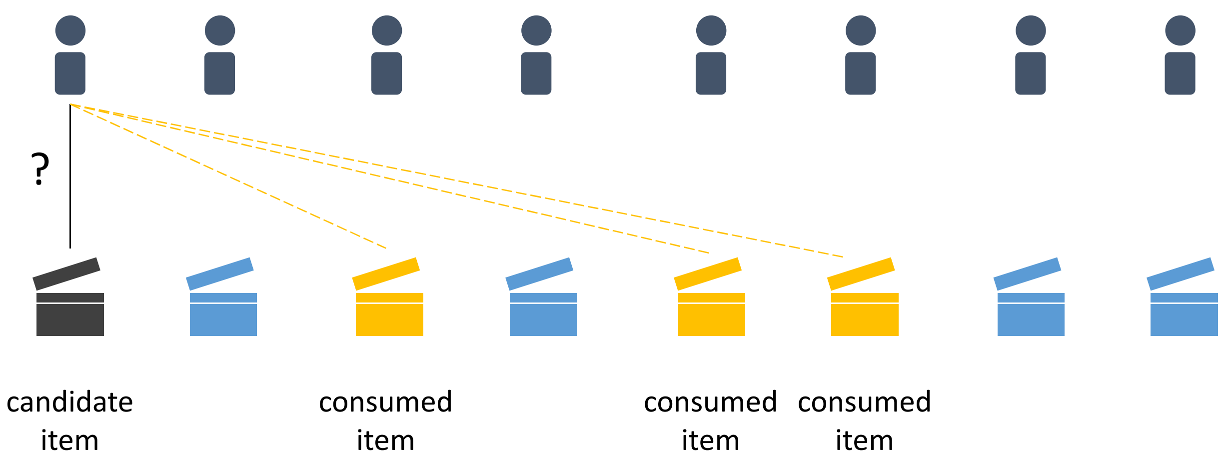
- Factor model conveniently allows separately treating different aspects
- We observe changes in:
  - Rating scale of individual users $b_u(t)$
  - Popularity of individual items $b_i(t)$
  - User preferences $p_u(t)$

$$r_{u,i}(t) = \mu + b_u(t) + b_i(t) + p_u(t)^\top q_i$$

- Design guidelines
  - Items show slower temporal changes
  - Users exhibit frequent and sudden changes
  - Factors $p_u(t)$ are expensive to model
  - Gain flexibility by heavily parameterizing the functions

Neighborhood (Similarity)-based MF

- Assumption: user's previous consumed items reflect her taste
- Derive unknown ratings from those of "similar" items (item-item variant)



Neighborhood based MF modeling: SVD++

$$\hat{r}_{u,i} = b_{u,i} + q_i^\top \Big(p_u + |N(u)|^{-\frac{1}{2}} \sum_{j \in N(u)} y_j \Big)$$

need to estimate rating of user u for item i

bias

item vector

user vector

Set of user previously rated items

item vector for neighborhood similarity

- Each item has two latent vectors
  - The standard item vector q_i
  - The vector y_i when it is used for estimating the similarity between the candidate item and the target user

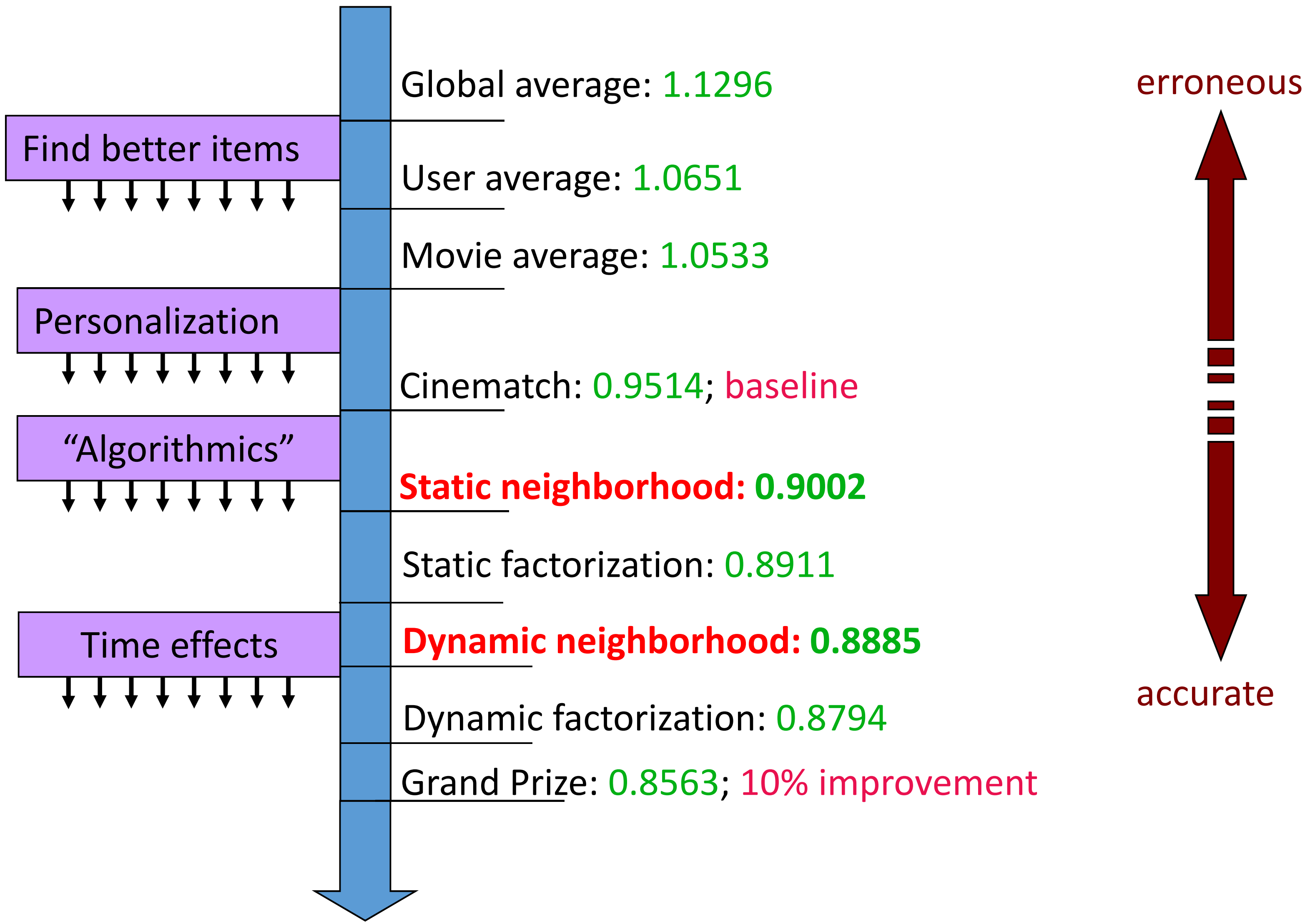
Koren, Yehuda. "Factorization meets the neighborhood: a multifaceted collaborative filtering model." *KDD*, 2008.

Netflix Prize

- An open competition for the best collaborative filtering algorithm for movies
  - Began on October 2, 2006.
  - A million-dollar challenge to improve the accuracy (RMSE) of the Netflix recommendation algorithm by 10%

- Netflix provided
  - Training data: 100,480,507 ratings:
  - 480,189 users x 17,770 movies.
  - Format: <user, movie, date, rating>

- Two popular approaches:
  - Matrix factorization
  - Neighborhood



Temporal neighborhood model delivers same relative RMSE improvement (0.0117) as temporal factor model (!)


NETFLIX
2009
DATE: 09.21.09
PAY TO THE ORDER OF: BellKor's Pragmatic Chaos
$1,000,000 00
AMOUNT: ONE MILLION 00/100
FOR: The Netflix Prize
Reed Hastings
The Netflix Prize

Feature-based Matrix Factorization

$$\hat{y} = \mu + \Big(\sum_j b_j^{(g)}\gamma_j + \sum_j b_j^{(u)}\alpha_j + \sum_j b_j^{(i)}\beta_j\Big) + \Big(\sum_j p_j\alpha_j\Big)^\top \Big(\sum_j q_j\beta_j\Big)$$

- Regard all information as features
  - User id and item id
  - Time, item category, user demographics etc.
- User and item features are with latent factors

Tianqi Chen et al. Feature-based matrix factorization. arXiv:1109.2271
http://svdfeature.apexlab.org/wiki/images/7/76/APEX-TR-2011-07-11.pdf
Open source: http://svdfeature.apexlab.org/wiki/Main_Page

Factorization Machine

$$\hat{y}(\mathbf{x}) = w_0 + \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \langle v_i, v_j \rangle x_i x_j$$

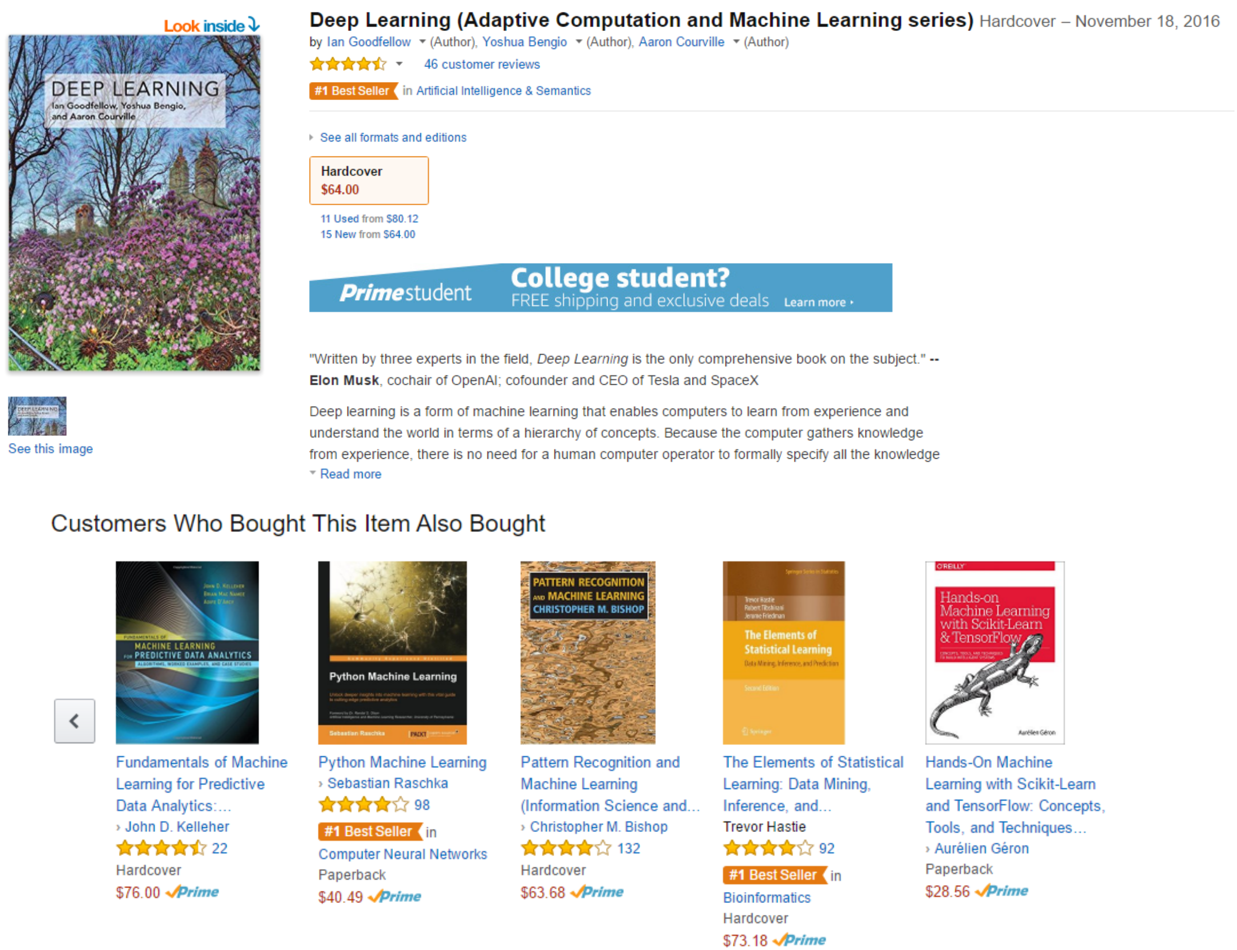
- One-hot encoding for each discrete (categorical) field
- One real-value feature for each continuous field
- All features are with latent factors
- A more general regression model

Steffen Rendle. Factorization Machines. ICDM 2010
http://www.ismll.uni-hildesheim.de/pub/pdfs/Rendle2010FM.pdf
Open source: http://www.libfm.org/

Beyond Rating Prediction

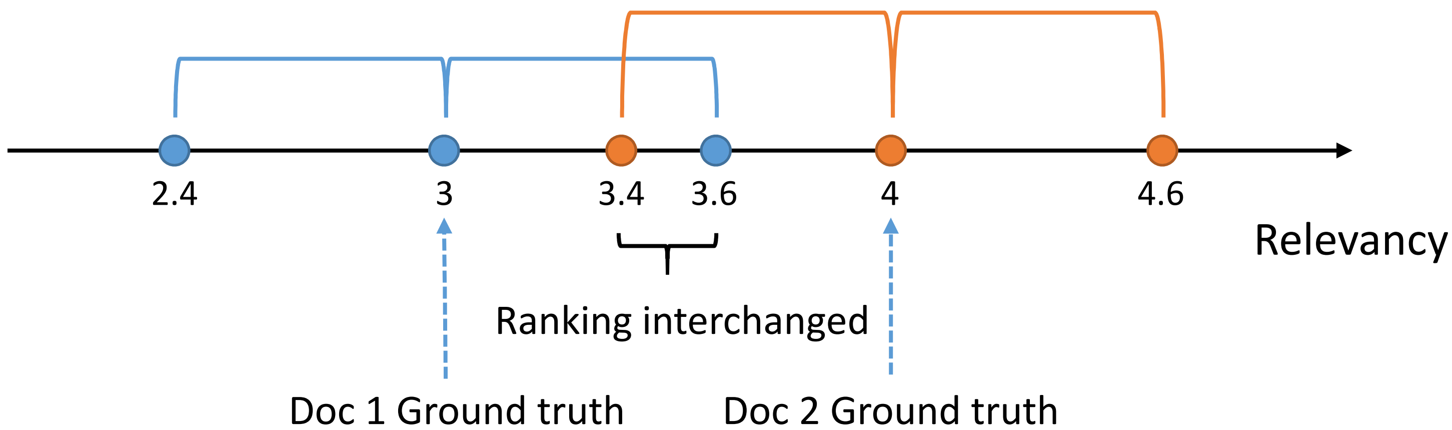
LambdaRank CF

Recommendation is always rendered by ranking



Rating Prediction vs. Ranking

- Rating prediction may not be a good objective for top-N recommendation (i.e. item ranking)



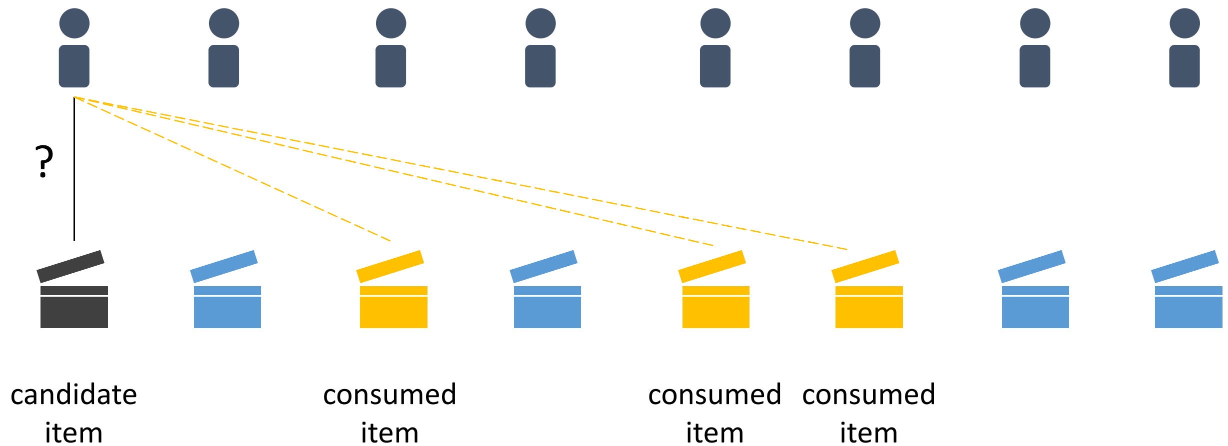
- Same RMSE/MAE might lead to different rankings

Learning to Rank in Collaborative Filtering

- Previous work on rating prediction can be regarded as pointwise approaches in CF
  - MF, FM, kNN, MF with temporal dynamics and neighborhood information etc.
- Pairwise approaches in CF
  - Bayesian personalized ranking (BPR)
- Listwise approaches in CF
  - LambdaRank CF, LambdaFM

Implicit Feedback Data

- No explicit preference, e.g. rating, shown in the user-item interaction
  - Only clicks, share, comments etc.



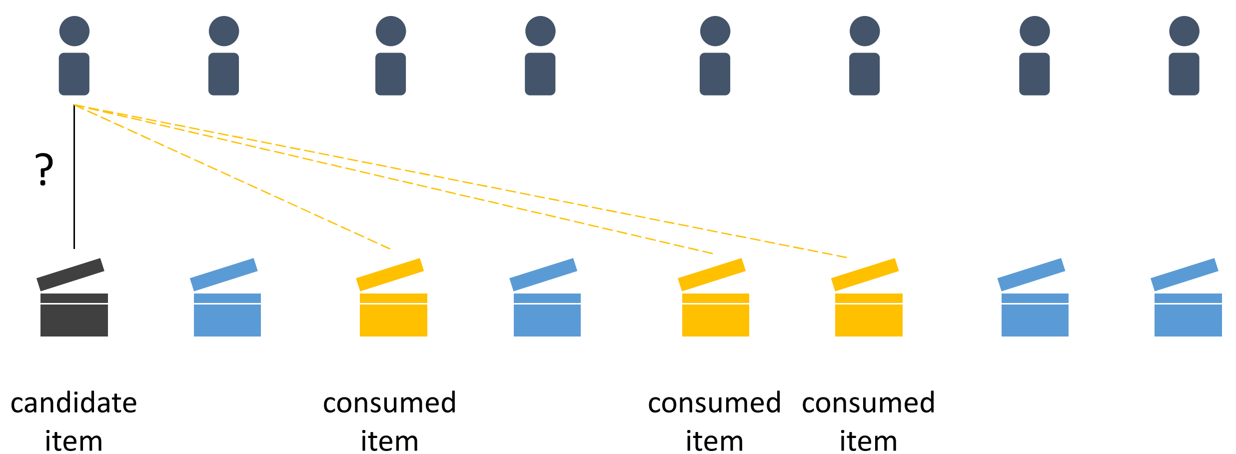
Bayesian Personalized Ranking (BPR)

- Basic latent factor model (MF) for scoring

$$\hat{r}_{u,i} = \mu + b_u + b_i + p_u^\top q_i$$

- The (implicit feedback) training data for each user *u*

$$D_u = \{\langle i, j \rangle_u | \underbrace{i}_{\text{consumed item}} \in I_u \wedge \underbrace{j}_{\text{unconsumed item}} \in I \backslash I_u\}$$



Rendle, Steffen, et al. "BPR: Bayesian personalized ranking from implicit feedback." *UAI*, 2009.

Bayesian Personalized Ranking (BPR)

- Loss function on the ranking prediction of <*i,j*>$_u$

$$\mathcal{L}(\langle i,j\rangle_u) = z_u \cdot \frac{1}{1+\exp(\hat{r}_{u,i}-\hat{r}_{u,j})}$$

Normalizer   Inverse logistic loss

- Gradient

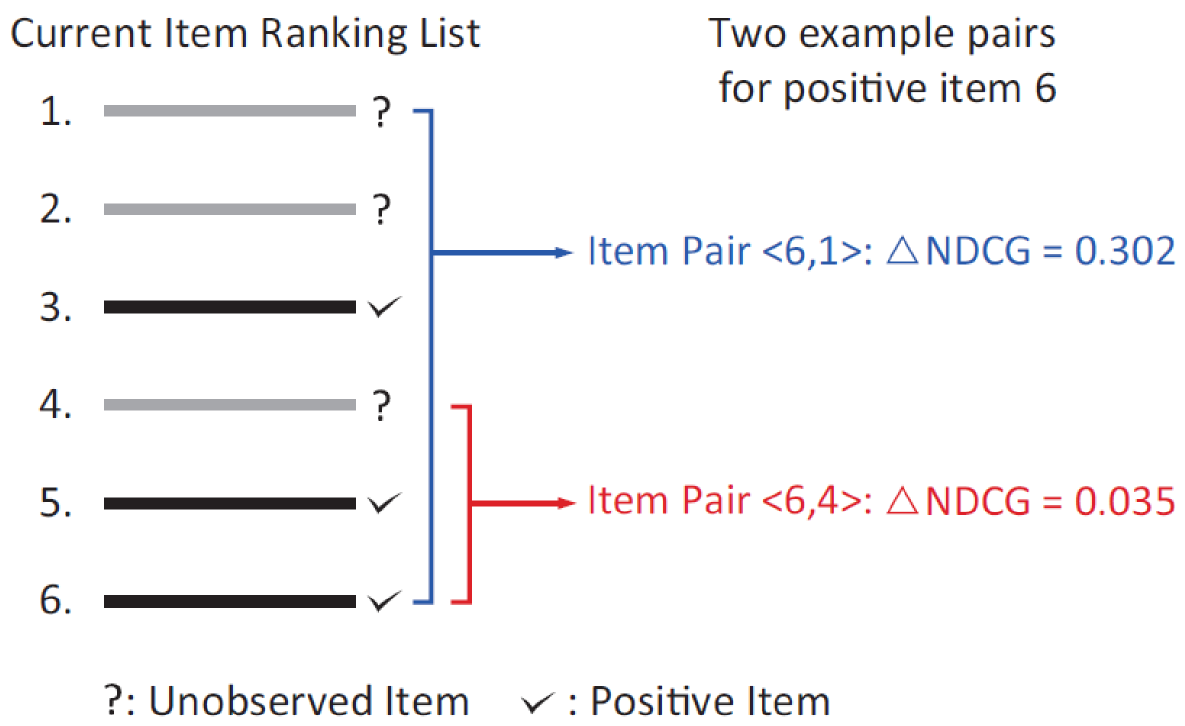
$$\frac{\partial \mathcal{L}(\langle i,j\rangle_u)}{\partial \theta} = \frac{\partial \mathcal{L}(\langle i,j\rangle_u)}{\partial(\hat{r}_{u,i}-\hat{r}_{u,j})}\frac{\partial(\hat{r}_{u,i}-\hat{r}_{u,j})}{\partial \theta}$$

$$\equiv \lambda_{i,j}\left(\frac{\partial \hat{r}_{u,i}}{\partial \theta} - \frac{\partial \hat{r}_{u,j}}{\partial \theta}\right)$$

Rendle, Steffen, et al. "BPR: Bayesian personalized ranking from implicit feedback." *UAI*, 2009.

LambdaRank CF

- Use the idea of LambdaRank to optimize ranking performance in recommendation tasks



Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

Recommendation vs. Web Search

- Difference between them
  - Recommender system should rank all the items
    - Usually more than 10k
  - Search engine only ranks a small subset of retrieved documents
    - Usually fewer than 1k
- For each training iteration, LambdaRank needs the model to rank all the items to get $\Delta NDCG_{i,j}$, super large complexity

Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

LambdaRank CF Solution

- Idea: to generate the item pairs with the probability proportional to their lambda

$$\frac{\partial \mathcal{L}(\langle i,j\rangle_u)}{\partial \theta} = f(\lambda_{i,j},\zeta_u)\Big(\frac{\partial \hat{r}_{u,i}}{\partial \theta} - \frac{\partial \hat{r}_{u,j}}{\partial \theta}\Big)$$

$$f(\lambda_{i,j},\zeta_u) \equiv \lambda_{i,j}\Delta NDCG_{i,j}$$

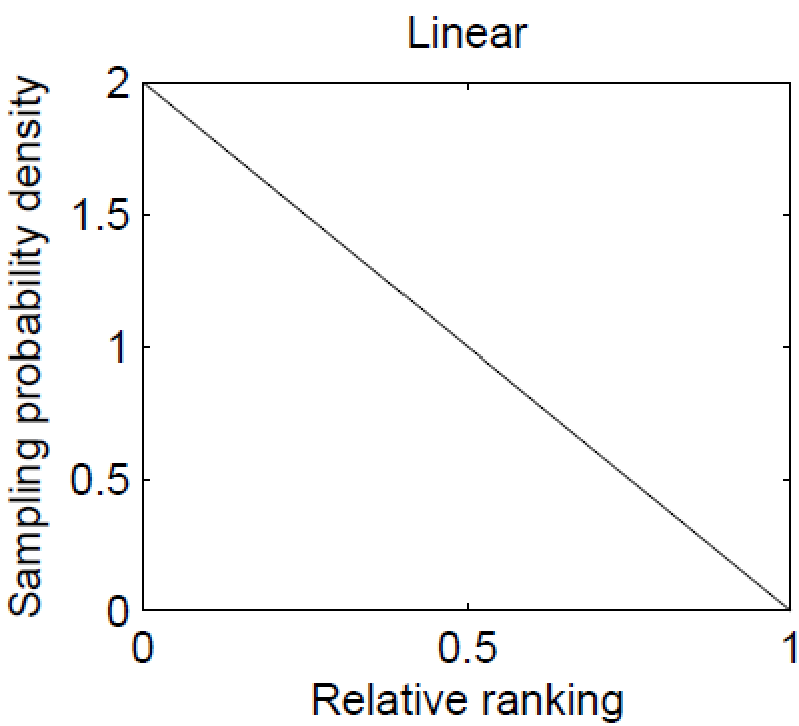
$$p_j \propto f(\lambda_{i,j},\zeta_u)/\lambda_{i,j}$$

- $x_i \in [0,1]$ is the relative ranking position
  - 0 means ranking at top, 1 means ranking at tail

Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

Different Sampling Methods

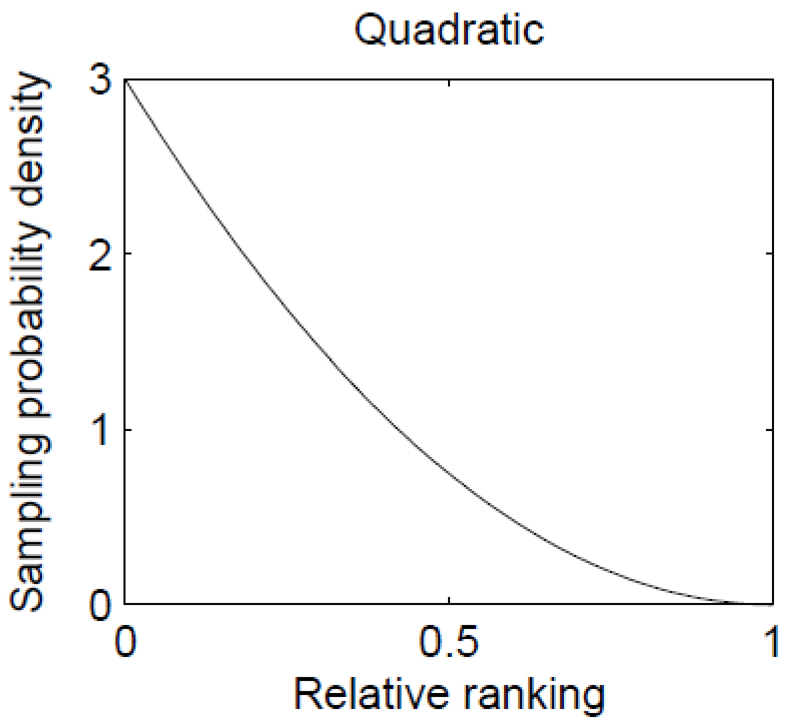
- For each positive item, find 2 candidate items, then choose the one with higher prediction score as the negative item.



Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

Different Sampling Methods

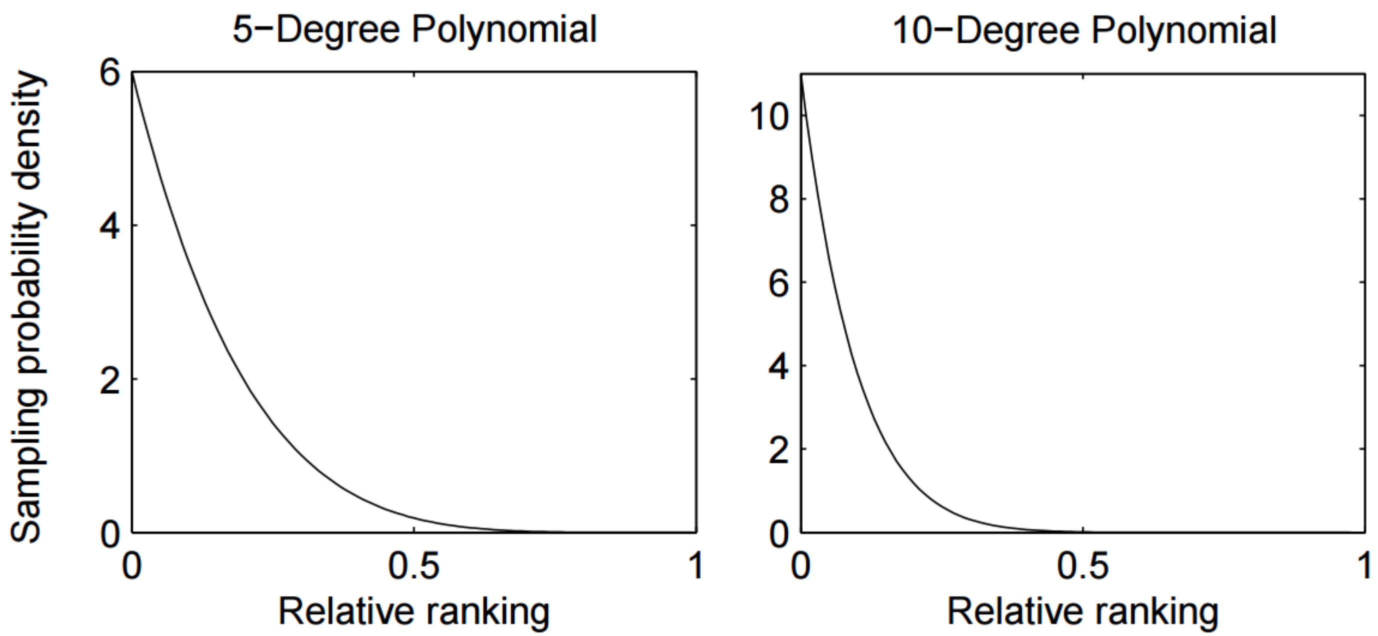
- For each positive item, find **3** candidate items, then choose the one with the **highest** prediction score as the negative item.



Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

Different Sampling Methods

- For each positive item, find ***k*** candidate items, then choose the one with the **highest** prediction score as the negative item.



Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

Experiments on Top-N Recommendation

- Top-N recommendation on 3 datasets

Dataset	Netflix	Yahoo! Music	Last.fm
Users	480,189	1,000,990	992
Items	17,770	624,961	961,417
Ratings	100,480,507	262,810,175	19,150,868

- Performance (DNS is our LambdaCF algorithm)

Netflix

	P@5	P@10	NDCG@5	NDCG@10	MAP
BPR	0.3826	0.3272	0.2052	0.2017	0.1403
DNS	0.4708	0.4012	0.2906	0.2887	0.2036
Impv.	23.1%*	22.6%*	41.6%*	43.1%*	45.1%*

Yahoo! Music

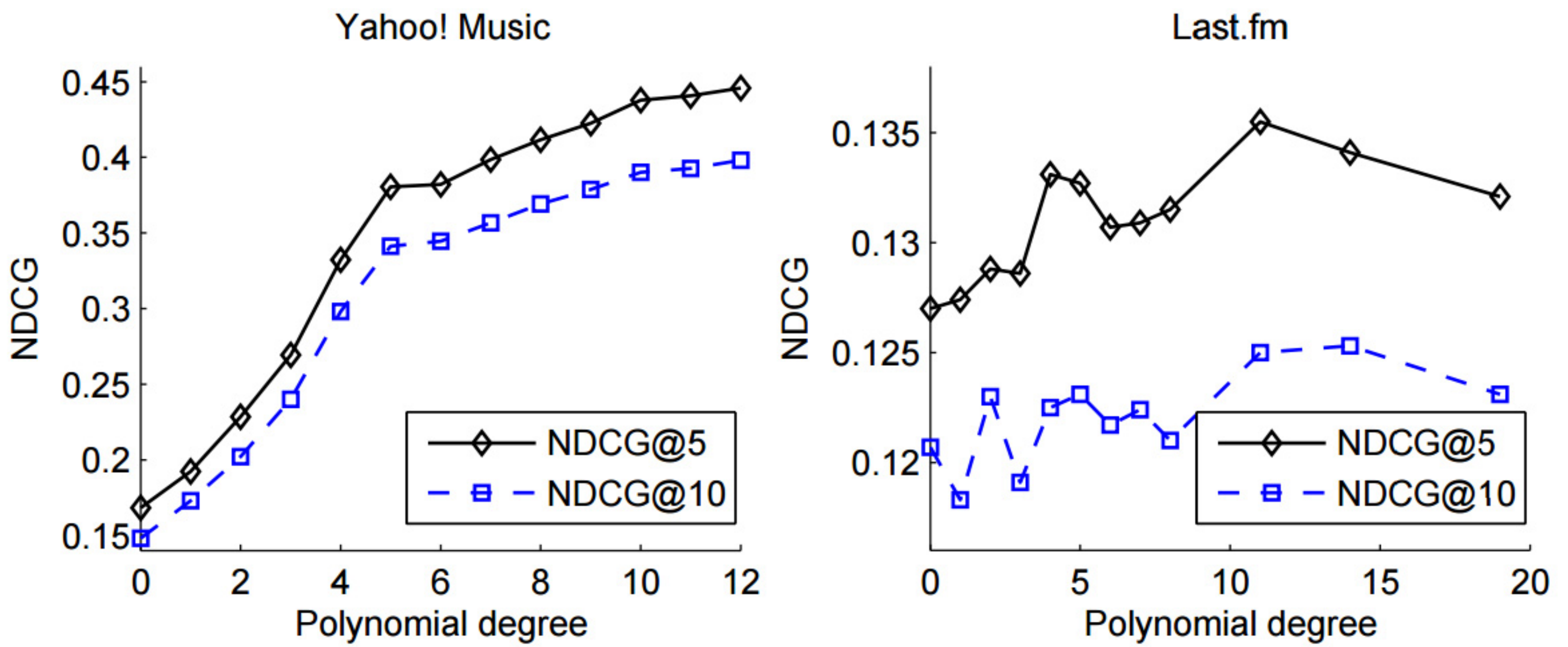
	P@5	P@10	NDCG@5	NDCG@10	MAP
BPR	0.1588	0.1359	0.1683	0.1481	0.0615
DNS	0.4243	0.3671	0.4458	0.3981	0.1644
Impv.	167.2%*	170.1%*	164.9%*	168.8%*	167.3%*

Last.fm

	P@5	P@10	NDCG@5	NDCG@10	MAP
BPR	0.1231	0.1168	0.1270	0.1207	0.0221
DNS	0.1323	0.1202	0.1355	0.1250	0.0223
Impv.	7.5%*	2.9%	6.7%*	3.6%	0.9%

Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

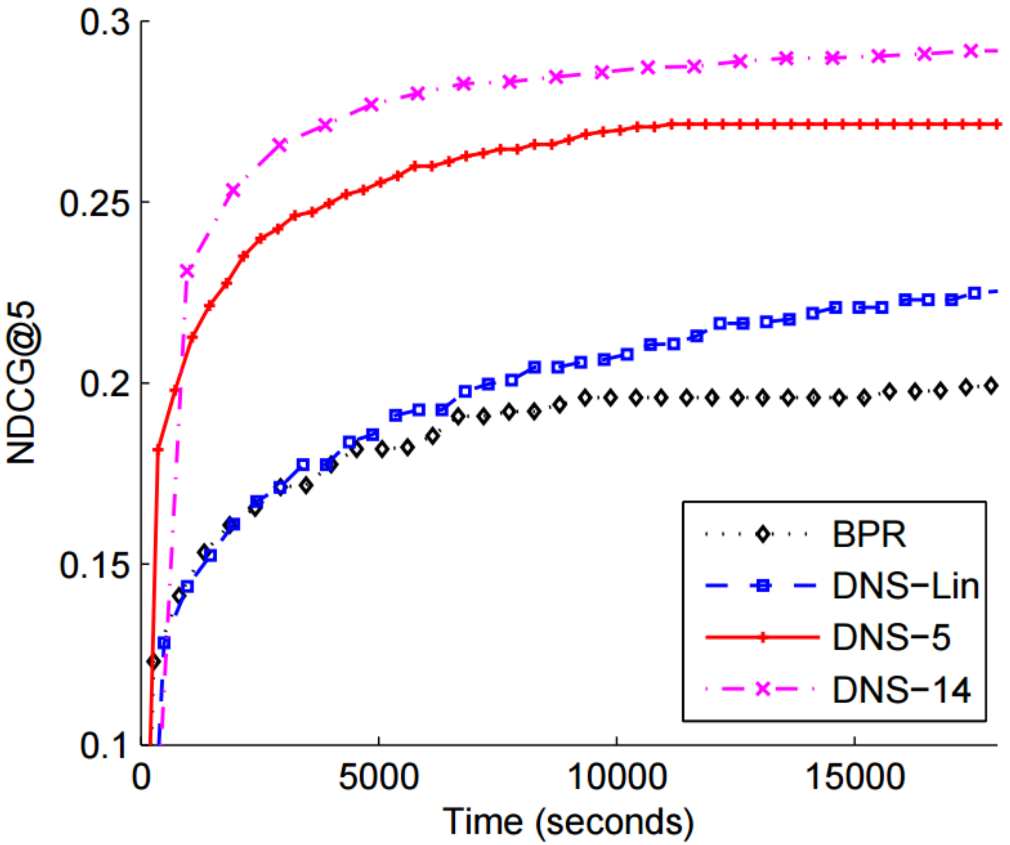
More Empirical Results



- NDCG performance against polynomial degrees on Yahoo! Music and Last.fm datasets

Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.

More Empirical Results



Performance convergence against training time on Netflix.

Zhang, Weinan, et al. "Optimizing top-n collaborative filtering via dynamic negative item sampling." *SIGIR*, 2013.