Real-Time Bidding Algorithms for Performance-Based Display Ad Allocation

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ABSTRACT

We describe a real-time bidding algorithm for performance-based display ad allocation. A central issue in performance display advertising is matching campaigns to ad impressions, which can be formulated as a constrained optimization problem that maximizes revenue subject to constraints such as budget limits and inventory availability. The current practice is to solve the optimization problem offline at a tractable level of impression granularity (e.g., the placement level), and to serve ads online based on the precomputed static delivery scheme. Although this offline approach takes a global view to achieve optimality, it fails to scale to ad delivery decision making at an individual impression level. Therefore, we propose a real-time bidding algorithm that enables fine-grained impression valuation (e.g., targeting users with real-time conversion data), and adjusts value-based bid according to real-time constraint snapshot (e.g., budget consumption level). Theoretically, we show that under a linear programming (LP) primal-dual formulation, the simple real-time bidding algorithm is indeed an online solver to the original primal problem by taking the optimal solution to the dual problem as input. In other words, the online algorithm guarantees the offline optimality given the same level of knowledge an offline optimization would have. Empirically, we develop and experiment with two real-time bid adjustment approaches to adapting to the non-stationary nature of the marketplace: one adjusts bids against real-time constraint satisfaction level using control-theoretic methods, and the other adjusts bids also based on the historical bidding landscape statistically modeled. Finally, we show experimental results with real-world ad serving data.

1. INTRODUCTION

In this paper we consider the problem of performance-based display ad allocation. The goal is to match campaigns (demand-side) to ad impressions (supply-side) such that the total revenue from a publisher's perspective (also referred to as yield) is maximized, while satisfying the constraints primarily imposed by campaign budget and supply inventory availability. In performance-based display advertising, there are two major pricing models, namely cost-per-click (CPC) and cost-per-action (CPA). At an abstract level, advertisers place their bids in the form of CPC or CPA prices on ad impression opportunities from publishers, an ad serving system makes delivery decisions based on advertisers bids, estimated click-through rate (CTR) or conversion rate (CVR), and relevant constraints, and then advertisers only pay publishers for performance-based metrics, i.e., clicks or conversions on their ads. Without loss of generality, we will focus our discussion on an abstract one-sided marketplace of advertisers, with only one publisher and no intermediaries (e.g., ad networks). We further assume that an ad delivery system is optimizing revenue on behalf of the publisher, as well as use CPC campaigns for illustration purpose.

Under an unconstrained environment, particularly if advertisers have unlimited budget, the revenue-optimal allocation mechanism is to simply assign each impression to the campaign with highest expected revenue per impression (eCPI = CTR $\times$ CPC). However, it can be trivially shown that this naïve mechanism is suboptimal under a constrained setting where demand-side constraints exist. The current practice is to solve a constrained optimization problem offline (see Section 6 for a formal account), and use the static optimal allocation to guide ad delivery. The constrained optimization is formulated as a linear programming (LP) problem, where the total revenue is maximized with respect to impression allocations subject to linearized constraints. Although an offline optimization takes a global view to achieve revenue optimality at a certain aggregated level, this approach has considerable limitations as follows.

1. An offline global optimization can only be solved at a tractable level of impression granularity. In the current formulation of performance optimization for one large display network for instance, impressions are allocated at the campaign-placement level, there are about 1M decision variables and 0.5M constraints. Impressions within each node of this granularity are treated homogeneous in term of valuation, therefore finer-grained impression data cannot be leveraged. Some of these impression-level opportunities are known to be differentiating, such as user historical behavior [5].

2. Even if some constraints are inherently at an aggre-

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gated level, such as campaign-level budget constraints and placement-level traffic volume, the offline optimization will still soon become intractable as the number of advertisers and publishers becomes very large, as in ad exchanges [14].

3. Since the offline optimization can only creates a static allocation scheme, it lacks a natural way to adapt to the marketplace dynamics. In particular, when the distribution from which winning bids are drawn (bidding landscape) changes and the campaign goal is over or under delivered as a consequence, an offline algorithm has no mechanism to adjust accordingly [7].

Motivated by the limitations of the current approach and the emergence of ad exchanges for clearing performance display ads [14], we develop a real-time bidding algorithm that enables fine-grained or even individual impression valuation while preserving the offline revenue optimality. The real-time bidding algorithm is not only elegant but also well-grounded on the duality principle. It turns out that the simple online algorithm solves the original offline optimization problem as the primal by taking the optimal solution to the dual as input, i.e., the bid adjustment term. The dual optimal thus bears natural interpretation, that is, the shadow price of an extra unit of the primal constraint. With this theoretical foundation from a static perspective, the bidding algorithm requires two key predictive components to account for the dynamic nature of a marketplace.

1. Impression valuation is a function that returns the expected value of an impression to a bidder (advertiser or campaign) given relevant impression-level and demand-side data, which takes the form:

$$\text{eCPI} = \text{CTR} \times \text{CPC}, \tag{1}$$

where eCPI stands for expected cost per impression, and shall be the optimal bid in an unconstrained environment. The CPC price is a constant w.r.t. a given campaign. The critical task is to estimate the click-through rate (CTR):

$$\text{CTR} = p(\text{click}|\text{campaign,impression}), \tag{2}$$

where impression in the conditional side encodes relevant impression-level data including user, and campaign encodes demand-side data. The problem of estimating CTR is reasonably well-studied [1, 5, 8]. For a real-time bidding algorithm, a CTR model shall leverage fine-grained and real-time predictor variables, usually high-dimensional and sparse, and perform online prediction efficiently, e.g., in 10's milliseconds [14]. While we will not focus on building a CTR predictive model, we follow the art and use a regression-based model.

2. Real-time bid adjustment is to adjust the value-based bid according to (1) the bidding landscape so far, and (2) the current constraint satisfaction level. We propose an additive bid adjustment as follows.

$$\text{bid} = \text{eCPI} - \alpha, \tag{3}$$

where α is the bid adjustment term. Intuitively, when the bidding landscape becomes more competitive (i.e., the probability of winning becomes lower given a same bid), and there are more impressions to acquire to satisfy constraints, α should be smaller (or even negative) to bid higher. Theoretically, the additive form can be derived from a primal-dual formulation. Specifically, we develop two real-time bid adjustment approaches (see Section 5 for the update formulae):

(a) A control-theoretic approach that adjusts the bid against constraint satisfaction level based on some error function. We present two forms of optimality control: (1) a proportional-integral controller, and (2) an exponential form.

(b) A statistical approach that estimates the distribution from which winning bids are drawn, and then derives the bid adjustment to achieve a desired winning probability, also based on the constraint satisfaction level.

We shall note that the impression valuation function as in Eq. (1) is with respect to an advertiser and is not necessarily the same as the revenue function of a publisher (think of a second-price auction). To abstract from the complication of what auction to run, which itself is an open research issue [14], we assume in our discussion a first-price auction run in a one-sided marketplace of advertisers, without loss of the generality of the basic approach. In this case, Eq. (1) is equivalent to the revenue function. This simplifying assumption is removed in Section 6 when we develop more general bidding algorithms, e.g., from an ad network point of view and in a second-price auction. Indeed, in a standard auction in a one-sided market as we assume, the revenue equivalence theorem states that first-price, second-price and several other auctions will all yield identical expected revenue in Bayesian equilibrium [10].

2. BACKGROUND

The primary theoretical foundation upon which the online bidding algorithms are grounded is LP duality. We briefly introduce the concepts fundamental to the derivation that follows. The standard form of a LP problem, referred to as a primal problem, is:

$$\begin{aligned} \max_x \ & c^\top x \\ \text{s.t. } & Ax \le b, \\ & x \ge 0. \end{aligned} \tag{4}$$

Here x is a vector of n variables, c is a vector of n coefficients, and $c^\top x$ is the objective function. A is a $m \times n$ matrix of coefficients, b is a vector of m coefficients, and $Ax \le b$ are the constraints, as well as the non-negative variable constraints. A x is feasible if it satisfies all constraints, and is optimal if it gives the maximal value of the objective function among all feasibles.

The standard form can be equivalently converted into an augmented form by introducing a non-negative slack variable for each of the m constraints to replace inequalities with equalities in the constraints. If the optimal value of a slack variable is zero in the augmented form, the corresponding inequality constraint in the stand form becomes tight in its optimal solution.

The primal problem can be converted into a dual problem:

$$\begin{aligned} \min_y \ & b^\top y \\ \text{s.t. } & A^\top y \ge c, \\ & y \ge 0. \end{aligned} \tag{5}$$

Here y is a vector of m variables corresponding to the m constraints in the primal, and $A^\top y \geq c$ are the n constraints corresponding to the n primal variables. The weak duality theorem states that the dual problem provides an upper bound to the objective function of the primal, i.e., $c^\top x \leq b^\top y, \forall$ feasible x, y. The strong duality theorem states that if the primal has an optimal solution x^*, then the dual also has an optimal solution y^*, such that $c^\top x^* = b^\top y^*$. Obviously, the primal and the dual are symmetric.

With the primal-dual formulation, we now have a choice of solving either the primal or the dual first (offline), according to the problem structure at hand (e.g., dimensionality and generality); and then derive (possibly online) an optimal solution to one (the primal) when an optimal solution to the other (the dual) is known. There is a very useful necessary condition for optimality that facilitates this idea.

Theorem 1. The complementary slackness theorem states: Suppose that

1. $x = (x_1, \ldots, x_n)^\top$ is a primal feasible,
2. $z = (z_1, \ldots, z_n)^\top$ is the corresponding dual slack,
3. $y = (y_1, \ldots, y_m)^\top$ is a dual feasible, and
4. $w = (w_1, \ldots, w_m)^\top$ is the corresponding primal slack.

Then x and y are optimal for their respective problems if and only if:

1. $x_i z_i = 0, \forall i$, and
2. $y_j w_j = 0, \forall j$.

The complementary slackness condition is a special case of the more general Karush-Kuhn-Tucker (KKT) conditions in convex optimization.

3. PERFORMANCE DISPLAY OPTIMIZATION: A LP FORMULATION

To derive the basic algorithmic form for online bidding, and to establish its optimality, we begin by formulating the basic performance display ad optimization as a LP. In this basic setting, impressions are valuated and allocated individually, and the demand-side constraints (e.g., budget limits) are given in terms of impression delivery goal. This formulation shall capture all theoretical essences, and practical nuances are discussed in Section 6. Let us first define the following notations:

1. i indexes n impressions, and j indexes m campaigns;
2. p_{ij} denotes CTR of impression i assigned to campaign j, q_j denotes CPC for campaign j, and $v_{ij} = p_{ij} q_j$ is the eCPI of such assignment;
3. g_j is the impression delivery goal for campaign j;
4. x_{ij} is the decision variable indicating whether impression i is assigned to campaign j ($x_{ij} = 1$) or not ($x_{ij} = 0$).

We formulate the following LP as the primal:

$$\begin{aligned} \max_x \ & \sum_{i,j} v_{ij} x_{ij} \\ \text{s.t. } \forall j, \ & \sum_i x_{ij} \leq g_j, \\ \forall i, \ & \sum_j x_{ij} \leq 1, \\ & x_{ij} \geq 0. \end{aligned} \tag{6}$$

The dual problem is then:

$$\begin{aligned} \min_{\alpha,\beta} \ & \sum_j g_j \alpha_j + \sum_i \beta_i \\ \text{s.t. } & \forall i, j, \ \alpha_j + \beta_i \geq v_{ij}, \\ & \alpha_j, \beta_i \geq 0. \end{aligned} \tag{7}$$

It is important to note that since an impression is by nature indivisible, the primal should actually be an integer programming problem, by adding the integer variable constraint $x_{ij} \in \{0, 1\}$. However, as we show more rigorously in Section 4, the optimal solution to the primal LP is indeed integral, given some nice structure of this basic setting, specifically the constraint matrix is totally unimodular (TU). Even if the LP optimal solution is no longer integral, e.g., using budget constraints directly instead of impression delivery goal, the LP relaxation will not pose any significant problems in practice. For impression i, a fractional optimal x_{ij} can be thought of as a probabilistic assignment, i.e., assign impression i to campaign j with probability x_{ij}. Assume that there are sufficiently large amount of impressions identical to i (to the extent a practical system can tell), then not only will the probabilistic assignment yield an integral solution, but also optimal for the LP. As a consequence, we can safely use algorithms to ensure an integral solution.

The dual variables have natural interpretations: α_j is the economic value of acquiring an additional goal impression for campaign j (e.g., by increasing budget), and β_i is the economic value of procuring an additional impression i for the publisher (e.g., by attracting more visits).

We wish to derive an online algorithm to solve the primal LP, thus obtaining a delivery scheme $x_{ij}, \forall i, j$. One may choose to solve the primal directly, as in the current practice of display ad allocation. In this case, the number of variables (x_{ij}) to learn is $n \times m$. On the other hand, solving the dual only requires $n + m$ variables (α_j, β_i), orders of magnitude dimensionality reduction. The Occam's razor principle favors parsimonious models as the dual. The problems remaining are: (1) how to derive an optimal solution to the primal from the optimal solution to the dual, in an online fashion; and (2) how to account for the non-stationary nature of the distribution of impression arrivals. The bidding algorithm in Section 4 addresses the first problem, and the bid adjustment in Section 5 addresses the second problem.

4. A REAL-TIME BIDDING ALGORITHM

Consider an online setting, where each impression i arrives from a stream, and a real-time ad serving decision needs to be made, i.e., the assignment of $x_{ij}, \forall j$. For each incoming impression i, we will add m decision variables $x_{ij}, \forall j$ and one constraint $\sum_j x_{ij} \leq 1$ to the primal, one decision variable β_i and m constraints $\alpha_j + \beta_i \geq v_{ij}, \forall j$ to the dual. Let us for now assume that the optimal $\alpha_j, \forall j$ are given. The following online algorithm uses the complementary slackness theorem to assign variables such that the offline optimality is preserved.

Algorithm 1: The basic real-time bidding algorithm

Input: $q_j, g_j, \alpha_j, \forall j$
Output: $x_{ij}, \beta_i, \forall i, j$

begin
  $G \leftarrow \emptyset$;
  foreach *impression* i *from a stream* **do**
    $p_{ij} = p(\text{click}|i, j), \forall j$;
    $v_{ij} \leftarrow p_{ij} q_j, \forall j$;
    $j^* \leftarrow \text{argmax}_{j \notin G} (v_{ij} - \alpha_j)$;
    if $(v_{ij^*} - \alpha_{j^*}) > 0$ **then**
      $x_{ij^*} \leftarrow 1$;
      $x_{ij} \leftarrow 0, \forall j \neq j^*$;
      $\beta_i \leftarrow v_{ij^*} - \alpha_{j^*}$;
      if $\sum_{i'} x_{i'j^*} = g_{j^*}$ **then**
        $G \leftarrow G \cup j^*$;
      end
    end
    $\alpha_j \leftarrow \texttt{UpdateAlpha}(\alpha_j), \forall j$;
  end
end

The basic idea conveyed by Algorithm 1 is the following. For an incoming impression i, each campaign j valuates it with $v_{ij} = p(\text{click}|i, j)q_j$, and then bids on it with $v_{ij} - \alpha_j$. The winning bidder with positive bid gets the impression, and goal-achieved campaigns $j \in G$ will withdraw from future auctions. Here β_i is for bookkeeping the economic value of impression i and is not critical to auction. The bid adjustment function $\alpha_j \leftarrow \text{UpdateAlpha}(\alpha_j)$ is to adapt α_j's to the current goal delivery level and bidding environment. Under a stationary impression arrival assumption, it reduces to an identity function. We will implement the non-stationary version in Section 5. From an auction perspective, more interestingly, α_j can be interpreted as the minimum profit (revenue minus cost) campaign j requires, and β_i shall be proportional to the floor price demanded by the publisher who sells impression i.

We now establish the offline optimality of this basic online bidding algorithm under a stationary α assumption.

Theorem 2. The optimality of the online bidding algorithm. Given the optimal bid adjustment $\alpha_j, \forall j$, a first-price auction with the following design guarantees the offline optimality.

1. Every campaign j bids on an impression i with the amount $v_{ij} - \alpha_j$;

2. Only active campaigns, i.e., $\sum_{i'} x_{i'j} < g_j$, can participate;

3. The impression is assigned if and only if the highest bid is positive.

PROOF. Since a feasible solution exists (e.g., trivially set all variables to zero) and the objective function is bounded, there exist optimal solutions. For each impression i, the winning bidder is $j^* = \text{argmax}_{j \notin G} (v_{ij} - \alpha_j)$.

First, if the highest bid $(v_{ij^*} - \alpha_{j^*}) > 0$, impression i must be assigned; otherwise, the primal constraint $\sum_j x_{ij} < 1$ is slack, and the corresponding dual variable $\beta_i = 0$, thus the dual constraint $\alpha_{j^*} + \beta_i \geq v_{ij^*}$ contradicts. Then assign i to some j', so $x_{ij'} = 1$, and $\alpha_{j'} + \beta_i = v_{ij'}$. If $j' \neq j^*$, $(v_{ij^*} - \alpha_{j^*}) > (v_{ij'} - \alpha_{j'}) = \beta_i$ assuming no ties in bidding, thus the constraint $\alpha_{j^*} + \beta_i \geq v_{ij^*}$ contradicts. Therefore, the optimal solution contains $x_{ij^*} = 1$ and $x_{ij} = 0, \forall j \neq j^*$.

Second, if $(v_{ij^*} - \alpha_{j^*}) < 0$, the impression i should be ignored; otherwise, the tightened dual constraint $\alpha_j + \beta_i = v_{ij}$ corresponding to $x_{ij} = 1$ contradicts, since $\beta_i \geq 0$. If $(v_{ij^*} - \alpha_{j^*}) = 0$, there are multiple optimal solutions.

Third, the constraints $\sum_i x_{ij} \leq g_j, \forall j$ are satisfied by maintaining an active campaign set $\neg G$.

Finally, an impression with a positive highest bid must be all assigned (even in a divisible sense) to the highest bidder j^*; otherwise, if some losing bidder $j' \neq j^*$ gets assigned some non-zero impression (e.g., 0.2 impression), then the corresponding dual constraint tightened $\alpha_{j'} + \beta_i = v_{ij'}$, and hence contradicts the highest-bid proposition $(v_{ij'} - \alpha_{j'}) = \beta_i > (v_{ij^*} - \alpha_{j^*})$ assuming no ties in bidding. More formally, we can rewrite the primal constraints as in Eq. (6) in the standard matrix form:

$$\begin{bmatrix} I_m & I_m & \cdots & I_m \\ U_1 & U_2 & \cdots & U_n \end{bmatrix} \times \begin{bmatrix} x_{11} \\ \vdots \\ x_{1m} \\ x_{21} \\ \vdots \\ x_{nm} \end{bmatrix} \leq \begin{bmatrix} g_1 \\ \vdots \\ g_m \\ 1 \\ \vdots \\ 1 \end{bmatrix}. \tag{8}$$

Here I_m is a $m \times m$ identity matrix, and there are n of them horizontally stacked. $U_i, \forall i$ is a $n \times m$ matrix with ones on the ith row and zeros elsewhere. The constraint matrix A is then a $(n + m) \times (nm)$ matrix. Now we can verify that A is totally unimodular (TU). A unimodular matrix is a square integer matrix with determinant 1 or -1, and a TU matrix is a matrix for which every square non-singular submatrix is unimodular. It is known that if A is TU and b is integral, the LP of the standard form as in Eq. (4) has integral optima. □

5. BID ADJUSTMENT

One simplifying assumption we made in the basic algorithm is that the distribution from which impressions are drawn is stationary, which means given sufficient historical data we can learn the optimal bid adjustment $\alpha_j, \forall j$ a priori by solving the dual offline. In practice, however, since the marketplace is dynamic, impression arrival is non-stationary, e.g., seasonality changes of supply. On the other hand, demand-side valuation is also a non-stationary process, e.g., old campaigns expire and new campaigns begin. These non-stationarities will violate the complementary slackness condition, and hence the historically-optimal α_j's will not be optimal for future. Our approach is to initialize α_j's with the offline optimal solution to the dual formulated with historical data, and then update α_j's online to accommodate supply-side dynamics and demand-side constraint satisfaction level, through control-theoretic or statistical methods.

5.1 Control-theoretic Bid Adjustment

A simple control design is based on classical control theory, particularly we will use a proportional-integral (PI) controller [2], a commonly-used form of the more generic proportional-integral-derivative (PID) controller. It is known that, in the absence of knowledge of the underlying process, a PID controller is the best controller [3]. Formally, let t denote time, $r_j(t)$ and $r'_j(t)$ are the desired and observed probabilities of winning bids, respectively, tracked at time t; and $e_j(t) = r_j(t) - r'_j(t)$ is the error measured at time t. The PI controller takes the following form:

$$\alpha_j(t+1) \leftarrow \alpha_j(t) - k_1 e_j(t) - k_2 \int_0^t e_j(\tau) d\tau. \tag{9}$$

Here k_1 is called proportional gain, and k_2 is called integral gain, both are tuning parameters. In practice, time t needs not to be tracked instantaneously, for both online computational efficiency and the discrete nature of impression arrival. Instead, let $t \in [1, \dots, T]$ indexes sufficiently small time intervals, where T is the number of intervals within the entire duration of online bidding; and one only updates α_j's once after each interval.

Another even simpler control approach is inspired by Waterlevel [4], an online and fast approximation algorithm originally designed for resource allocation problems, such as delivering placement-reserved display ads. The Waterlevel-based update formula is:

$$\alpha_j(t+1) \leftarrow \alpha_j(t) \exp\left(\gamma(x_j(t)/g_j - 1/T)\right), \forall j, \tag{10}$$

where $x_j(t)$ denotes the number of impressions won by campaign j during time interval t; and the exponent factor γ is a tuning parameter that controls how fast the algorithm responds to the error measured as $x_j(t)/g_j - 1/T$. If the initial α_j's (e.g., solved from the offline dual) are indeed optimal for future runs, we want γ to be zero. Notice though, in the error term $x_j(t)/g_j - 1/T$ we assume for clarity a uniform impression stream over time intervals. This assumption is not critical since it can be easily removed by adding a time-dependent prior. Moreover, the Waterlevel-based update has a nice chaining property:

$$\begin{aligned}\alpha_j(t+1) &= \alpha_j(t) \exp\left(\gamma(x_j(t)/g_j - 1/T)\right) \\ &= \alpha_j(t-1) \exp\left(\gamma\left(\sum_{\tau=t-1}^{t} x_j(\tau)/g_j - 2/T\right)\right) \\ &= \dots \\ &= \alpha_j(1) \exp\left(\gamma\left(\sum_{\tau=1}^{t} x_j(\tau)/g_j - t/T\right)\right).\end{aligned} \tag{11}$$

5.2 Model-based Bid Adjustment

The basic idea of our model-based approach is drawn from modern control theory [9], where a mathematical model of the state of the system (the bidding marketplace in our case) is utilized to produce a control signal (the bid adjustment α_j's in our case). More formally, we postulate a parametric distribution $\mathcal{P}$ on the winning bids as follows.

$$w \sim \mathcal{P}(\theta), \tag{12}$$

where θ is the model parameter. We use the generic form since an appropriate parametric choice should be empirically justified by data, and likely domain-dependent. Several reasonable choices are a log-normal distribution [7] and a Gaussian distribution on the square-root of winning bids [13], but neither can handle negative bids naturally. In our additive form of bid adjustment as in Eq. (3), a negative bid $b_{ij} = v_{ij} - \alpha_j < 0$ would mean that the bidder cannot fulfill its minimal margin by acquiring impression i, thus reveals a hidden part of the entire value book of the marketplace. We note the PDF as $f(w;\theta)$, the CDF as $F(w;\theta)$, and the inverse CDF as $F^{-1}(p;\theta)$. The MLE of the distribution parameter θ is derived from sufficient statistics of historical winning bids $\{w\}$, which can be readily updated online, e.g., the first and second moments. With this statistical model of bidding landscape, we can derive the probability of winning by bidding with $b_{ij} = v_{ij} - \alpha_j$:

$$p(w \le b_{ij}) = \int_{-\infty}^{b_{ij}} f(w;\theta) dw = F(b_{ij};\theta). \tag{13}$$

It is unrealistic to assume that the winning bids for all impression i's follow a single distribution (more likely a mixture model), thus we will fit a distribution $\mathcal{P}(\theta)$ for a group of homogeneous impressions, e.g., from a placement. In practice, we align a $\mathcal{P}(\theta)$ with the impression granularity level at which both the supply-side and demand-side constraints the system wishes to enforce (more on this in Section 6). For now let us focus on homogeneous impressions.

We wish to link the learned winning probability with future bidding behavior to achieve the delivery goal. Suppose that r_j is the expected or desired probability of winning the remaining impressions by campaign j to meet its goal g_j. It is tempting to just bid with $b_{ij} = F^{-1}(r_j;\theta)$ on future impression i's. However, this purely goal-based approach fails to use feedback explicitly to control future bids, thus losing the advantages that a so-called closed-loop controller (e.g., a PID controller) would have, including stability and robustness to model uncertainty. In other words, the purely goal-driven approach does not learn from the errors made by past bidding. We now propose a model-based controller that addresses this limitation, while leveraging the knowledge learned about the bidding landscape. The bid adjustment formula is as follows.

$$\alpha_j(t+1) \leftarrow \alpha_j(t) - \gamma\left(F^{-1}(r_j(t)) - F^{-1}(r'_j(t))\right), \forall j, \tag{14}$$

where $r_j(t)$ and $r'_j(t)$ are desired and observed winning probabilities, respectively, measured at time t. The multiplicative factor γ is a tuning parameter that controls the rate at which an update responds to errors. Compared with classic approaches, a model-based approach does not directly operate on measured errors; instead it transforms an error signal (winning probability error), through a compact model ($\mathcal{P}(\theta)$), to a control signal (updated α_j).

It is also worth noting that in the absence of a good parametric distribution, a non-parametric model can also be used in practice. One needs to maintain an empirical CDF as a two-way lookup table ($F(w;D)$ and $F^{-1}(p;D)$) for online inference.

6. PERFORMANCE DISPLAY OPTIMIZATION: A PRACTICAL FORMULATION

We have developed the basic algorithmic form in Algorithm 1, and established its optimality given the stationary impression arrival assumption. In the basic LP formulation, constraints are encoded as impression delivery goal, and impressions are valuated and assigned individually. We now formulate the LP problem directly with business constraints, primarily demand-side budget limits and supply-side inventory availability; and then discuss major practical aspects that a real-world system shall take into consideration. Let us first update the following notations:

1. i now indexes n impression groups (e.g., placements), impressions within one group are regarded as indistinguishable, and hence yield a same CTR estimation given a campaign;
2. g_j is the budget cap for campaign j;
3. h_i denotes the impression availability constraint or forecast for group i;
4. x_{ij} now denotes the number of impressions from group i allocated to campaign j;
5. w_i denotes the (traffic acquisition) cost per impression from group i, e.g., the second price in a Vickrey auction.

Notice that we make CTR prediction and supply constraint at the same resolution, i.e., per impression group. This is critical to avoid the so-called cream-skimming problem [11]. If CTR prediction is finer-grained than supply constraint for instance, an optimization will always assign impressions to higher-CTR opportunities within each impression group, which is obviously unrealistic. Also, we introduce the cost term w_i to generalize the yield optimization to other players, e.g., an ad network or demand-side platform participating into a second-price auction. The primal LP now becomes:

$$\begin{aligned} \max_{x} \ & \sum_{i,j} (v_{ij} - w_i) x_{ij} \\ \text{s.t. } \forall j, \ & \sum_{i} v_{ij} x_{ij} \le g_j, \\ \forall i, \ & \sum_{j} x_{ij} \le h_i, \\ & x_{ij} \ge 0. \end{aligned} \tag{15}$$

And the dual problem is:

$$\begin{aligned} \min_{\alpha,\beta} \ & \sum_{j} g_j \alpha_j + \sum_{i} h_i \beta_i \\ \text{s.t. } \forall i,j, \ & v_{ij}\alpha_j + \beta_i \ge v_{ij} - w_i, \\ & \alpha_j, \beta_i \ge 0. \end{aligned} \tag{16}$$

By a similar derivation as in Section 4, the following auction design would approach offline revenue optimality:

1. Every campaign j bids on an impression from group i with the amount $v_{ij}(1 - \alpha_j)$;
2. Only active campaigns, i.e., $\sum_i v_{ij} x_{ij} < g_j$, can participate;
3. The impression is assigned only if the highest bid is greater than the cost w_i, which is implied if w_i only contains the second price.

The bid adjustment term α_j can now be interpreted as the minimum profit margin (profit divided by revenue) a campaign requires. The cost term w_i is constant with respect to each auction.

Since the primal constraint matrix A in Eq. (15) is no longer TU with the valuation terms v_{ij}'s introduced, the primal optimal is not necessarily integral. As we demonstrated in Section 4, however, the LP relaxation is not a concern in practice. One can solve the primal x^*_{ij}'s, and then stochastically assign impressions from group i to campaign j with probability $x^*_{ij} / \sum_{j'} x^*_{ij'}$, referred to as proportional allocation. Many current practices follow this primal-based approach [4, 6].

As one wishes to valuate impressions at a very fine-grained level, the primal will soon become intractable (recall the cream-skimming problem). We propose a dual-based approach as follows. First, the dual as in Eq. (16) is solved offline using historical data. The offline optimal α_j's are then used as the initial bid adjustment terms for future bidding. In online bidding, a centralized ad delivery system (e.g., owned by the publisher) with perfect information (e.g., the historically optimal α_j's) bids on behalf of all advertisers, and adjusts α_j's near real-time based on observed constraint satisfaction level and bidding landscape, through approaches described in Section 5. The dual has lower dimensionality to start with, i.e., $n + m$ model parameters instead of nm, thus conceived to have a better generality. What we seek from the dual is actually a partial solution, that is, α_j's only, which only causes linear complexity of the offline solver. Furthermore, since we will adjust α_j's dynamically anyway, a good approximation may suffice.

A practical algorithm shall take into consideration the following aspects. First, to avoid multimodality, one distribution density $\mathcal{P}(\theta)$ is estimated for one impression group that exhibits sufficient homogeneity, e.g., a Gaussian distribution on winning bids per group $w \sim \mathcal{N}(\mu_i, \sigma_i^2)$ to allow for negative bids. Since the purpose of the statistical model is to derive a constraint-based bid from the desired winning probability based upon budget limit, the budget constraint g_j needs to be given at the same impression group level at which the distribution is fit. However, constraints such as budget cap are naturally given at the campaign level across all impression groups. This issue can be addressed both offline and online. One can start with distributing offline a campaign-level goal across impression groups with a prior proportion, and then synchronize online these campaign-group level subgoals. More interestingly, the other part of the dual solution, namely β_i's, shall guide this subgoal synchronization. If some β_i becomes positive and larger, the inventory availability from impression group i becomes scarce, which suggests one shall move some delivery subgoals from group i to those with ampler resources and bid less aggressively for group i (think of a reverse auction where impression groups bid for campaigns). Here we focus on pre-distributed campaign-group level goals g_j's.

Second, we consider a fully observed exchange where the winning bid $w_{s'}$ is revealed after each auction. A partially observed setting differs in the penalty for a bidder to learn the distribution of winning bids, i.e., w_i will contain some exploration cost [7].

Third, the algorithm shall be implemented as an online listener, where each impression s arrives from a stream $\mathcal{S}$ one at a time. For online computational efficiency, only minimally required statistics are updated real-time, specifically impression assignments, sample sizes, cumulative valuation, and moments for model parameter θ. Bid adjustment, along with its relevant parameter estimation, is performed near real-time, only once after each time interval t; so does campaign budget capping. These near real-time updates may involve expensive computations such as inversing a CDF.

Finally, the estimated CTR p_{sj}, along with eCPI v_{sj} and bid b_{sj}, is made dependent upon individual impressions s. This is only for a bidder to randomize its bids, but not for cream skimming. The expected value of a randomized CTR should still be the group-level CTR, i.e., $\mathbb{E}(p_{sj}) = p_{ij}, \forall s \in i$. Bid randomization has two advantages: (1) bidding ties can be naturally broken (see the optimality theorem and its proof in Section 4), and tie-breaking is especially important in reserved ad allocation where valuations may largely overlap; and (2) one can extend the optimal auction theory [15] to show that the revenue-optimal mechanism for an ad network is to randomize its bids (e.g., by a uniform distribution) within a certain range [14].

7. EXPERIMENTS

In this section, we discuss our empirical evaluation with the application of implicit targeting for performance display advertising [1], using real-world ad serving data. The goal of implicit targeting is to optimize revenue by leveraging fine-grained impression-level opportunities, such as user signals. Compared with explicit targeting, such as behavioral targeting [5], the audience to target is not explicitly defined, but instead is identified via impression-level valuation. The current approach to performance ad optimization is to solve a LP offline for determining ad assignment at the (placement, campaign) level, and to use a static proportional allocation scheme for online ad delivery. In implicit targeting, we wish to perform the LP optimization at the (placement, user, campaign) level, and to solve it online. Here a user is a user-level feature vector that encodes signals relevant to predicting clicks. By distinguishing among users within a same placement, one shall expect higher revenue than treating them identical. In comparison with the current offline approach, the proposed online bidding algorithm for implicit targeting not only makes impression-level valuation feasible, but also yields substantial computational advantage. Formally, we formulate the problem of implicit targeting as a LP similar to Eq. (15), with the following specializations.

1. Campaign-level budget constraints are given as impression delivery goals g_j's.
2. An impression group i is defined as a (placement, user) tuple, at which level both CTR prediction p_{ij} and inventory control $\sum_j x_{ij} \le h_i$ are performed.
3. The cost term w_i will be zero, since impressions are from owned inventory.

7.1 Data, Methodology and Metrics

We aim to empirically answer the following questions:

1. Whether the online bidding algorithm can obtain the offline revenue optimality, given the optimal bid adjustment α_j?
2. How different bid adjustment approaches perform, their optimality approximation, and control-theoretic properties?
3. How significant is the initial value of α_j, and to solve it offline with historical data?

A proprietary data set of ad serving for one of the most trafficked placements is obtained from a large display network. There are on average 20M impressions served on each day, and four CPC campaigns involved. The raw log is processed into examples with the schema: (timestamp, placement, user, campaign, clicks, impressions). For each example, let us denote the timestamp as t, the (placement, user) pair as i, the campaign as j, the clicks as $c_{ij}(t)$, and the impressions as $x_{ij}(t)$. When t is tracked instantaneously, an example shall correspond to a single impression assignment. For computational efficiency in our experiments, we trace timestamp in seconds, and hence the composite key (t, i, j) uniquely identifies an example. The impressions $x_{ij}(t), \forall t, i, j$ are the allocation results of the current production system based on a (placement, campaign) level LP optimization, thus aggregating impressions for each campaign j gives the impression delivery goal $g_j = \sum_{t,i} x_{ij}(t)$. The clicks $c_{ij}(t)$ are the ground-truth feedbacks, thus divided by impressions gives the empirical CTR. For simulating revenue generated by the proposed approach, we are particularly interested in the empirical CTR for each (placement, user, campaign) tuple marginalized over time, i.e., $p_{ij} = \frac{\sum_t c_{ij}(t)}{\sum_t x_{ij}(t)}$. One may choose to use time-dependent empirical CTR $p_{ij}(t) = \frac{c_{ij}(t)}{x_{ij}(t)}$ to simulate revenue, but that would lead to noisy estimation due to the sparsity of $x_{ij}(t)$. The sum of the products of clicks by CPC q_j is the actual revenue yielded by the current approach $y = \sum_{t,i,j} c_{ij}(t) q_j$.

The online bidding algorithm is implemented as follows, similar to Algorithm 1 but with practical considerations as discussed in Section 6. One first sorts examples by timestamp to form a stream. For each incoming example from the stream, one predicts CTR for each campaign $p'_{ij}, \forall j$, regardless of the actual assigned campaign. Active campaigns then bid with $b_{ij} = p'_{ij} q_j - \alpha_j$, where α_j is initialized as the dual optimal solved from historical data and updated online as bidding runs. The impressions associated with the example are re-assigned to the winning bidder $j^* = \mathrm{argmax}_j (b_{ij})$, i.e., $x'_{ij^*}(t) \leftarrow x_{ij}(t)$ if $b_{ij^*} > 0$; or ignored if $b_{ij^*} \le 0$,. By so simulating, the impression volume constraints $\forall i, \sum_j x_{ij} \le h_i$ are naturally satisfied, since all impressions already exist and one only changes the campaign assignments. The campaign-level goals are checked real-time after each auction, while α_j's are updated near real-time once after each hour, i.e., $T = 24$. The algorithm runs on a daily basis, and at the end of each day one will have a simulated impression allocation $x'_{ij}(t), \forall t, i, j$. The simulated revenue is computed as $y' = \sum_{t,i,j} x'_{ij}(t) p_{ij} q_j$, where p_{ij} is the empirical CTR instead of the predicted CTR. To evaluate the performance of different algorithms and bid adjustments, we use revenue lift defined as the ratio of simulated revenue to actual revenue:

$$\text{Revenue lift} = \frac{y'}{y} = \frac{\sum_{t,i,j} x'_{ij}(t) p_{ij} q_j}{\sum_{t,i,j} c_{ij}(t) q_j}. \tag{17}$$

By removing the CPC term, we can also report the overall CTR lift by the proposed approach:

$$\text{CTR lift} = \frac{\text{CTR}'}{\text{CTR}} = \frac{\sum_{t,i,j} x'_{ij}(t) p_{ij} / \sum_{t,i,j} x'_{ij}(t)}{\sum_{t,i,j} c_{ij}(t) / \sum_{t,i,j} x_{ij}(t)}. \tag{18}$$

A sensible implicit targeting algorithm shall have a revenue lift greater than one, and the offline optimal solution gives an upper-bound lift to any online algorithm.

7.2 Results

Let us start with verifying the optimality of the online bidding algorithm. We take one-day ad serving data, first solve both the primal and the dual LP offline to obtain the offline optimal revenue and α_j's, and then simulate the online bidding on the same data using the optimal α_j's without adjustment. As shown in Table 1, the online algorithm achieves nicely the offline revenue optimality. The small variance is because this particular implementation allocates in batch impressions sharing a same composite key (t, i, j) to save computation, which may violate inventory constraints marginally. Although CTR is not the optimizing objective, the online algorithm yields higher CTR lift than offline. This is because the online algorithm only assigns impressions with positive winning bids, and may disregard perceived non-monetizable ones; while the offline solver does not concern as long as the constraints are satisfied.

Table 1: Optimality of Online Bidding

Methods	Revenue lift	CTR lift
Offline optimal	1.4811	1.3180
Online bidding	1.4821	1.4913

We now evaluate different online bid adjustment approaches, specifically the model-based and the Waterlevel-based control mechanisms (shortcut as ModelBidder and Waterlevel-Bidder). Both uses historical optimal α_j's as initial values solved from the day preceding the testing day, and we test on four consecutive days. The rate factor γ is set to 0.01 for ModelBidder, and $0.24 = 0.01T$ for WaterlevelBidder. The results are shown in Table 2.

Table 2: ModelBidder vs. WaterlevelBidder

Revenue lift	Day1	Day2	Day3	Day4
Offline optimal	1.3656	1.4243	1.3652	1.4811
ModelBidder	1.3598	1.3936	1.3683	1.4817
WaterlevelBidder	1.2460	1.3923	1.3675	1.4681

Both bid adjustment methods perform very well in approaching offline revenue optimality, with above 90% offline optimal revenue lift achieved in all cases. The model-based adjustment does slightly better in revenue lift. It is also important to examine the control-theoretic properties of different adjustment algorithms, specifically the stability. Let us define the hourly delivery ratio of a campaign as the hourly winning impressions as a percentage of its daily delivery goal, and the reference ratio as the hourly impression arrivals as a percentage of the total daily impressions. A stable control strategy shall produce delivery ratios close to the reference ratio. We plot the delivery ratios of different adjustment methods against the reference ratio for one top campaign in terms of delivery goal on one day, as shown in Figure 1. The Waterlevel-based method shows superior stability since it directly operates on the error measured against the reference, while the model-based exhibits greater oscillations. On the other hand, using static optimal α_j does not react to the reference.

To investigate the significance of the initial value of α_j and the merit of solving it offline with historical data, we simulate the Waterlevel-based bid adjustment with different initial α_j values, namely, (1) zero α_j's, (2) historical optimal α_j's, and (3) offline optimal α_j's for the testing day. In reality, one cannot know offline optimal values of α_j's a priori. The results are shown in Table 3.

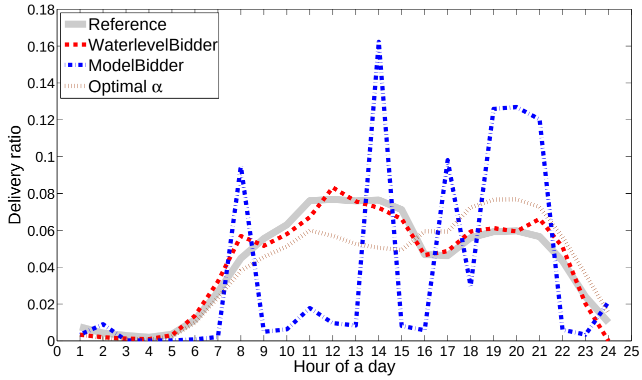

Figure 1: Hourly delivery ratio against reference

Table 3: Significance of Initial α_j

Revenue lift	Day1	Day2	Day3	Day4
Offline optimal	1.3656	1.4243	1.3652	1.4811
Zero α_j	1.3652	1.3984	1.3696	1.4819
Historical α_j	1.2460	1.3923	1.3675	1.4681
Optimal α_j	1.3557	1.3973	1.3655	1.4638

Although all different choices approximate offline revenue optimality, varying initial value of α_j makes no significant difference. We argue that this may be an artifact of offline simulation, where only offline data pre-filtered by the current production system could be obtained. The actual impression allocation in the data has already gone through constraint enforcement by the current delivery system that produces the data; and as a consequence, the delivery goal for the online bidding simulation will not be constraining enough to make α_j significant at the beginning. In fact as empirically shown, all campaigns only exhaust their delivery goals towards the end of a daily run. In reality, when campaign budgets become much more constraining relative to the impressions available from an open exchange, a good initial α_j will matter more.

To test the hypothesis of the pre-selection bias of the offline data and to fully reveal the significance of the initial α_j, we artificially constrain the campaign budget by a factor of $\lambda \in [0, 1]$, thus the campaign delivery goal g_j derived from the data becomes λg_j. We experiment with $\lambda = 90\%, 75\%, 50\%$ using the Waterlevel-based bid update, and the results support the hypothesis, as shown in Tables 4, 5, and 6. As campaign budget becomes smaller and delivery goal becomes more constraining, the performance of the zero-α_j initialization degrades considerably, while the historial or offline optimal α_j largely preserves the offline revenue optimality. When campaign budget is cut by 50%, a cold start with zero-α_j cannot even recover the revenue yielded by the current approach. These empirical results justify the cost of solving an offline dual LP to obtain a sensible α_j initialization, especially under a constraining budget.

8. CONCLUSIONS

Our contributions are a simple yet well-grounded real-time

Table 4: Constraining budget by 90%

Revenue lift	Day1	Day2	Day3	Day4
Offline optimal	1.3300	1.3930	1.3652	1.4814
Zero α_j	1.2587	1.2797	1.2562	1.3368
Historical α_j	1.1123	1.3795	1.2536	1.4507
Optimal α_j	1.3269	1.3824	1.3218	1.3406

Table 5: Constraining budget by 75%

Revenue lift	Day1	Day2	Day3	Day4
Offline optimal	1.2858	1.3540	1.3652	1.4813
Zero α_j	1.0796	1.0789	1.0671	1.0953
Historical α_j	1.1092	1.3431	1.3380	1.4599
Optimal α_j	1.2694	1.3526	1.3480	1.3658

Table 6: Constraining budget by 50%

Revenue lift	Day1	Day2	Day3	Day4
Offline optimal	1.2702	1.3248	1.3655	1.4815
Zero α_j	0.7315	0.7076	0.7300	0.7194
Historical α_j	1.2252	1.3181	1.1330	1.4792
Optimal α_j	1.2228	1.1277	1.3555	1.4425

bidding algorithm for performance display ad allocation, its theoretical revenue optimality guarantee, and practical approaches to adapting bids to market dynamics. We have provided a general framework of bidding in performance-based marketplace, where both impression valuation and constraints are taken into consideration to come up with an optimal bid. Further online experiments are desired to understand bid adjustment behavior even better, particularly to investigate the significance of several sources of model uncertainties including impression valuation and bidding landscape distribution. Finally, it will be of great interest to examine the equilibrium properties of the proposed bidding framework at an exchange, and particularly to compare the proposed bidding algorithms with other auction mechanisms with heterogeneous valuations [12, 13].

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10. REFERENCES

[1] D. Agarwal, R. Agrawal, R. Khanna, and N. Kota. Estimating rates of rare events with multiple hierarchies through scalable log-linear models. *Proceedings of the 16th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD 2010)*, pages 213–222, 2010.

[2] K. J. Astrom and R. M. Murray. *Feedback Systems: An Introduction for Scientists and Engineers.* Princeton University Press, 2008.

[3] S. Bennett. *A History of Control Engineering, 1930-1955.* IET, 1993.

[4] D. Charles, M. Chickering, N. R. Devanur, K. Jain, and M. Sanghi. Fast algorithms for finding matchings in lopsided bipartite graphs with applications to display ads. *Proceedings of the 11th ACM Conference on Electronic Commerce (EC 2010)*, 2010.

[5] Y. Chen, D. Pavlov, and J. F. Canny. Large-scale behavioral targeting. *ACM Conference on Knowledge Discovery and Data Mining (KDD 2009)*, pages 209–218, 2009.

[6] U. Feige, N. Immorlica, V. Mirrokni, and H. Nazerzadeh. A combinatorial allocation mechanism with penalties for banner advertising. *Proceeding of the 17th International World Wide Web Conference (WWW 2008)*, 2008.

[7] A. Ghosh, B. I. P. Rubinstein, S. Vassilvitskii, and M. Zinkevich. Adaptive bidding for display advertising. *Proceedings of the 18th International World Wide Web Conference (WWW 2009)*, pages 251–260, 2009.

[8] T. Graepel, J. Q. Candela, T. Borchert, and R. Herbrich. Web-scale Bayesian click-through rate prediction for sponsored search advertising in Microsoft's Bing search engine. *Proceedings of the 27th International Conference on Machine Learning (ICML 2010)*, pages 13–20, 2010.

[9] C. Kilian. *Modern Control Technology.* Delmar Cengage Learning, 2005.

[10] P. Klemperer. Auction theory: a guide to the literature. *Journal of Economic Surveys*, 13(3):227–286, 1999.

[11] J.-J. Laffont and J. Tirole. Optimal bypass and cream skimming. *American Economic Review*, 80(5):1042–1061, 1999.

[12] P. Lu, S. hua Teng, and C. Yu. Truthful auctions with optimal profit. *Workshop on Internet and Network Economics (WINE 2006)*, pages 27–36, 2006.

[13] V. Mirrokni, S. Muthukrishnan, and U. Nadav. Quasi-proportional mechanisms: prior-free revenue maximization. *arXivs*, 2009.

[14] S. Muthukrishnan. Ad exchanges: research issues. *Workshop on Internet and Network Economics (WINE 2009)*, pages 1–12, 2009.

[15] R. B. Myerson. Optimal auction design. *Mathematics of Operations Research*, 6(1):58–73, 1981.